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Strong convergence theorems of iterations for a pair of nonexpansive mappings in Banach spaces

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1. Introduction

Let E be a real Banach space and let C be a nonempty closed convex subset of E. Then a mapping T of C into itself is called nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. A mapping T of C into itself is called quasi-nonexpansive if the set $F(T)$ of fixed points of T is nonempty and $\|Tx - y\| \leq \|x - y\|$ for all $x \in C$ and $y \in F(T)$. For two mappings S, T of C into itself, Takahashi and Tamura [15] considered the following three iteration schemes :

$$x_{n+1} = \alpha_n S[\beta_n S x_n + (1-\beta_n)x_n] + (1-\alpha_n)T[\beta_n T x_n + (1-\beta_n)x_n], \quad (1)$$

$$x_{n+1} = \alpha_n S[\beta_n T x_n + (1-\beta_n)x_n] + (1-\alpha_n)[\gamma_n S x_n + (1-\gamma_n)x_n], \quad (2)$$

and

$$x_{n+1} = \alpha_n S[\beta_n T x_n + (1-\beta_n)x_n] + (1-\alpha_n)[\gamma_n T x_n + (1-\gamma_n)x_n] \quad (3)$$

for all $n \geq 1$, where $x_1 \in C$ and $\alpha_n, \beta_n, \gamma_n \in [0,1]$. In the case when $S = T$ and $\gamma_n = 0$ in (2) or (3), such an iteration scheme was considered by Ishikawa [5]; see also Mann [6]. Das and Debata [2] studied the strong convergence of the iterates $\{x_n\}$ defined by (2) or (3) in the case when $\gamma_n = 0$ and S, T are quasi-nonexpansive mappings in a strictly convex Banach space; see also Rhoades [10]. Tan and Xu [16] also discussed the weak convergence of the iterates $\{x_n\}$ defined by (2) or (3) in the case when $\gamma_n = 0$ and S, T are nonexpansive mappings in a uniformly convex Banach space which satisfies Opial's condition or whose norm is Fréchet differentiable. Later Takahashi and Kim[13] obtained strong and weak convergence theorems which are different from Tan and Xu [16].

In this paper, we deal with the strong convergence of iterates $\{x_n\}$ defined by (1), (2) and (3) in a strictly convex Banach space. First we prove that if we choose their suitable coefficients, the sequence $\{x_n\}$ defined by (1) converges strongly to an element of $F(S), F(T)$ or $F(S) \cap F(T)$. Further we study the strong convergence of iterates $\{x_n\}$ defined by (2) and (3). We note that there are many differences concerning the strong convergence of iterates defined by (2) and (3).