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ON MINIMAL *CR* SUBMANIFOLDS SATISFYING A CERTAIN CONDITION ON THE RICCI CURVATURE

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1. Introduction. We denote by $\bar{M}^m(c)$ a complex m-dimensional (real 2m-dimensional) Kaehlerian manifold of constant holomorphic sectional curvature 4c with Kaehlerian structure (J,g). Let M be a real n-dimensional Riemannian manifold isometrically immersed in $\bar{M}^m(c)$ with induced metric tensor field g. For any vector field X tangent to M, we put $JX = PX + FX$, where PX is the tangential part of JX and FX the normal part of JX. Then P is an endomorphism on the tangent bundle $T(M)$. If F vanishes identically, then M is called a *complex submanifold* of $\bar{M}^m(c)$, and if P vanishes identically, then M is callde an *anti-invariant submanifold* of $\bar{M}^m(c)$. A submanifold M of a Kaehlerian manifold $\bar{M}$ is called a *CR submanifold* of $\bar{M}$ if there exists a differentiable distribution $H : x \longrightarrow H_x \subset T_x(M)$ on M satisfying the following conditions:
(1) H is holomorphic, i.e., $JH_x = H_x$ for each $x \in M$, and
(2) the complementary orthogonal distribution $H^{\perp} : x \longrightarrow H_x^{\perp} \subset T_x(M)$ is anti-invariant, i.e., $JH_x^{\perp} \subset T_x(M)^{\perp}$ for each $x \in M$.

We denote by S the Ricci tensor of M. If M satisfies that $S(X,Y) = ag(X,Y)+bg(PX,PY)$, where a and b are constant, then M is called a ***pseudo-Einstein*** submanifold.

In [3] one of the present author proved that there are no Einstein real hypersurfaces of a complex projective space CP^m and classified the pseudo-Einstein real hypersurfaces of CP^m. This result was generalized by Cecil and Ryan [2] to the case that a and b are functions.

Moreover, Maeda [6] studied the Ricci tensor of a real hypersurface of a complex projective space.

On the other hand, one of the author [5] studied a compact minimal CR submanifold M of CP^m under the assumption that the Ricci tensor of M satisfies $S(X,X) \geq (n-1)g(X,X)+2g(PX,PX)$, and proved that M is a real projective space RP^n, or a complex projective space CP^n or a pseudo-Einstein real hypersurface $\pi\left(S^{(n+1)/2}\left(\sqrt{\tfrac{1}{2}}\right) \times S^{(n+1)/2}\left(\sqrt{\tfrac{1}{2}}\right)\right)$, where π denotes the projection with respect to the fibration $S^1 \longrightarrow S^{2m+1} \longrightarrow CP^m$.

The purpose of the present paper is to consider the problem on the Ricci tensor like that above without the assumption that M is compact.