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CONTROLLABILITY OF NONLINEAR FUNCTIONAL INTEGRO-DIFFERENTIAL SYSTEMS IN BANACH SPACE

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1. Introduction.

Controllability of linear and nonlinear systems represented by ordinary differential equations in finite-dimension space has been extensively studied. Several authors have extended the concept to infinite-dimension systems represented by evolution equations with bounded operators in Banach spaces(Ref.[4]) for Volterra integrodifferential systems, I and Kwun(Ref.[3]) studied the approximate controllability for delay Volterra systems with bounded linear operators in Banach space. The purpose of this paper is to study the controllability of abstract functional integrodifferential systems in Banach space by using the Schauder fixed point theorem and we give an example. The abstract functional integrodifferential equations are arised many physical phenomena.

2. Preliminaries.

Let X be a Banach space with norm $\|\cdot\|$ and let $C = C([-r,0],X)$ be the Banach space of continuous functions defined on $[-r,0], r>0$ with supermum norm $\|\cdot\|_C$. If x is continuous function from $[-r,T], T>0$ to X and $t\in[0,T]=J$, then x_t denotes the element of C given by $x_t(\theta)=x(t+\theta)$ for $\theta\in[-r,0]$. We consider the following abstract functional integrodifferential equation

$$
\begin{aligned}
\frac{d}{dt}x(t)+Ax(t) =& (Bu)(t)+\int_0^t [a(t,s)g(s,x_s)+h(t,s,x_s)]ds \\
(1) \qquad & + f(t,x_t), \quad t\in[0,T]=J \\
& x(t)=\phi(t), \quad -r\le t\le 0.
\end{aligned}
$$

where the state $x(\cdot)$ takes values in the Banach space X and the control function $u(\cdot)$ is given in $L^2(J;U)$, a Banach space of admissable control functions, with U a Banach space.
Here $-A$ is an infinitesimal generator of a strongly continuous semigroup $S(t), t\ge 0$ on X, and B is a bounded linear operator from U into X. The nonlinear functions $g: J\times C\to X$, $h: J\times J\times C\to X$, $f: J\times C\to X$ and the kernel $a: J\times J\to R$ (R denotes the set of real numbers) are continuous.

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