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REAL HYPERSURFACES IN COMPLEX HYPERBOLIC SPACE WITH η-RECURRENT SECOND FUNDAMENTAL TENSOR

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ABSTRACT. Recently, Hamada [4] has proved that there do not exist any real hypersurfaces in complex projective space $P_n(C)$ with recurrent second fundamental tensor. From this point of view, he introduce the notion of η-recurrent second fundamental tensor for real hypersurfaces in $P_n(C)$. In this paper we also consider the notion of η-recurrent second fundamental tensor for real hypersurfaces in complex hyperbolic space $H_n(C)$ and classified such kind of real hypersurfaces under the condition that the structure vector field ξ is principal.

1. Introduction

A complex $n(\geq 2)$-dimensional Kaehlerian manifold of constant holomorphic sectional curvature c is called a complex space form, which is denoted by $M_n(c)$. A complete and simply connected complex space form is a complex projective space $P_n(C)$, a complex Euclidean space C^n or a complex hyperbolic space $H_n(C)$, according as $c>0$, $c=0$ or $c<0$. The induced almost contact metric structure of a real hypersurface M of $M_n(c)$ is denoted by (ϕ, ξ, η, g).

There exist many studies about real hypersurfaces of $M_n(c)$. One of the first researches is the classification of homogeneous real hypersurfaces in a complex projective space $P_n(C)$ by Takagi [14], who showed that these hypersurfaces of $P_n(C)$ could be divided into six types which are said to be of type A_1, A_2, B, C, D, and E, and in [3] Cecil-Ryan and [7] Kimura proved that they are realized as the tubes of constant radius over Kaehlerian submanifolds if the structure vector field ξ is principal. Also Berndt [2] showed recently that all real hypersurfaces with constant principal curvatures of a complex hyperbolic space $H_n(C)$ are realized as the tubes of constant radius over certain submanifolds when the structure vector field ξ is principal. Nowadays in $H_n(C)$ they are said to be of type A_0, A_1, A_2, and B.

On the other hand, in [9] Kobayashi and Nomizu have introduced the notion of recurrent tensor field of type (r,s) on a manifold M with a linear connection. That is, a non-zero tensor field K of type (r,s) on M is said to be *recurrent* if there exists a 1-form α such that

$$\nabla K = K \otimes \alpha.$$

Moreover, they gave some geometric interpretation of a manifold M with recurrent curvature tensor in terms of holonomy group, see also [15].

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