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A note on Polysurface groups

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Introduction.

We shall define a group Γ to be a polysurface group of length n if there is a filtration $\{\Gamma_i\}_{0\leqq i\leqq n}$ of Γ such that

(1) Γ is torsion free,
(2) $1=\Gamma_0\subset\Gamma_1\subset\cdots\subset\Gamma_{n-1}\subset\Gamma_n=\Gamma$,
and
(3) for each i, $\Gamma_i\lhd\Gamma_{i+1}$ and Γ_{i+1}/Γ_i is the fundamental group of an orientable surface.

We call a group Γ a polysurface group without abelian factor if for each i, Γ_{i+1}/Γ_i is the fundamental group of an orientable surface with genus $\geqq 2$ and a polysurface group of length 1 a surface group.

In a series of his works ([J1],[J2]), F.E.A.Johnson has studied the smooth realization of polysurface group without abelian factor. Here a group Γ is called to be smoothly realizable when there exists a smooth manifold X_Γ whose fundamental group is Γ.

Being motivated by his works, we shall consider the following

Problem. *Does every polysurface group Γ embed as a discrete cocompact subgroup of a non-compact connected Lie group G without compact factor ?*

If a polysurface group Γ embeds as a discrete cocompact subgroup of a non-compact Lie group G, then Γ is realized as the fundamental group of smooth closed aspherical manifold $\Gamma\backslash G/K$, where K is a maximal compact subgroup of G.

In this note, we shall use the following notations.

1. A Lie group is assumed to be connected,non-compact,unless the contrary stated explicitly.
2. For a Lie group G, G° denotes the identity component.
3. $\tilde{X}$ denotes the universal covering space of X.
4. H^4, H^3 or H^2 denotes 4-dimensional, 3-dimensional or 2-dimensional hyperbolic space,respectively.
5. For a group G, $Z(G)$ denotes the center of G.
6. For a subgroup H of G, $N_G(H)$ or $C_G(H)$ denotes the normalizer or centralizer of H in G, respectively.
7. $\chi(X)$ denotes the Euler characteristic of X.

1. Preliminaries

We shall prove the following