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ON REAL HYPERSURFACES OF A COMPLEX PROJECTIVE SPACE WITH η-RECURRENT SECOND FUNDAMENTAL TENSOR

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0. Introduction.

Let M be an m-dimensional manifold with a linear connection Γ. A non zero tensor field K of type (r, s) on M is said to be *recurrent* if there exists a 1-form α such that $\nabla K = K \otimes \alpha$, where ∇ is covariant derivative with respect to Γ. We know the recurrent condition has a close relation to holonomy group in the sense of the following theorem (cf. [5] and [10]).

Theorem W. *œWe denote $L(M)$ be a bundle of frames of M and $T^r_s(\mathbf{R}^m)$ be a tensor bundle of type (r, s) over $\mathbf{R}^m$. Let $f : L(M) \to T^r_s(\mathbf{R}^m)$ be the mapping which corresponds to a given tensor field K of type (r, s). Then K is recurrent if and only if, for the holonomy bundle $P(u_0)$ through any $u_0 \in L(M)$, there exists a differentiable function $\psi(u)$ with no zero on $P(u_0)$ such that*

$$f(u) = \psi(u) f(u_0) \quad \text{for} \quad u \in P(u_0).$$

As a special case, K is parallel if and only if $f(u)$ is constant on $P(u_0)$.

We consider a real hypersurface M of real dimension $m = 2n - 1$ in a complex projective space $P_n(\mathbf{C})$, $n \geqq 2$ with Fubini-Study metric of constant holomorphic sectional curvature 4. Then M has an almost contact metric structure (ϕ, ξ, η, g) induced from the Kähler structure of $P_n(\mathbf{C})$. Many differential geometers have studied M by using the almost contact structure, for example [1], [2], [3], [4], [6] and [8]. It is well-known that there does not exist a real hypersurface M of $P_n(\mathbf{C})$ satisfying the condition that second fundamental tensor A of M is parallel. We have the following result under the weaker condition that the second fundamental tensor A is recurrent (cf. [7] and [9]).

Theorem 1. *There are no real hypersurfaces with recurrent second fundamental tensor of $P_n(\mathbf{C})$ on which ξ is a principal curvature vector.*

On the other hand Kimura and Maeda ([4]) introduced the notion of an η-parallel second fundamental tensor, which is defined by $g((\nabla_X A)Y, Z) = 0$