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WEAK TYPE INEQUALITY FOR POISSON MAXIMAL OPERATORS

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ABSTRACT. A necessary and sufficient condition for a certain maximal operator to be of weak type (p,q), $1\le p\le q<\infty$, is studied. This operator unifies various results about the Poisson integral operators cited in the literatures.

I. Introduction Consider the maximal operator

$$\mathcal{M}f(x,t)=\sup\{\frac{1}{|Q|}\int_Q |f(y)|\,dy : x\in Q \text{ and sidelength}(Q)\ge t\}.$$

It is well known that this maximal operator $\mathcal{M}$ controls Poisson integral defined by, for $x\in\mathbf{R}^n, t\ge 0$,

$$P(f)(x,t)=\int_{\mathbf{R}^n} f(y)P(x-y,t)dy,$$

where

$$P(x,t)=\frac{c_n t}{(|x|^2+t^2)^{\frac{n+1}{2}}}$$

is the Poisson kernel.

For a given positive measure ν on $\overline{\mathbf{R}^{n+1}_+}=\{(x,t):x\in\mathbf{R}^n, t\ge 0\}$, the problem under what conditions $\mathcal{M}$ is bounded from $L^p(\mathbf{R}^n)$ into $L^p(\overline{\mathbf{R}^{n+1}_+},\nu)$ and from $L^1(\mathbf{R}^n)$ into weak-$L^1(\overline{\mathbf{R}^{n+1}_+},\nu)$ was studied by several authors: Carleson[C] showed that $\mathcal{M}$ is bounded from $L^p(\mathbf{R}^n,dx)$ into $L^p(\overline{\mathbf{R}^{n+1}_+},d\nu)$ if and only if ν satisfies the Carleson condition

$$\sup_{x\in Q}\frac{\nu(\widetilde{Q})}{|Q|}\le C.$$

Later, Fefferman-Stein[FS] proved that $\mathcal{M}$ is bounded from $L^p(\mathbf{R}^n,w(x)dx)$ into $L^p(\overline{\mathbf{R}^{n+1}_+},d\nu)$ if

$$\sup_{x\in Q}\frac{\nu(\widetilde{Q})}{|Q|}\le Cw(x)\quad a.e.\quad x,$$

where $\widetilde{Q}=Q\times(0,l(Q)]$ if we denote $l(Q)$ the sidelength of Q. More recently, Ruiz[R] and Ruiz-Torrea[RT] unified various results concerning these problems.

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