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ENVELOPE OF HYPERHOLOMORPHY AND HYPERHOLOMORPHIC CONVEXITY

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Nôno investigated the hyperholomorphy of functions of quaternion variables in [13], [14], [15] and [16]. In the present paper, making use of his results [13] and [16] on series expansion and integral representation, for a Riemann's domain (Ω, φ) over $\mathbf{C}^2 \times \mathbf{C}^2$, we define the envelope $(\tilde{\Omega}, \tilde{\varphi})$ of hyperholomorphy of the domain (Ω, φ) and prove that the domain $(\tilde{\Omega}, \tilde{\varphi})$ is hyperholomorphically convex.

1. Hyperholomorphic function on a domain in $\mathbf{C}^2 \times \mathbf{C}^2$

The field $\mathcal{H}$ of quaternions

$$z = x_1 + ix_2 + jx_3 + kx_4, \qquad x_1, x_2, x_3, x_4 \in \mathbf{R} \tag{1}$$

is a four dimensional non-commutative $\mathbf{R}$-field generated by four base elements $1, i, j$ and k with the following non-commutative multiplication rule:

$$i^2 = j^2 = k^2 = -1, ij = -ji = k, jk = -kj = i, ki = -ik = j. \tag{2}$$

R. Fueter[7]-[8] and his school established the theory of quaternionic functions, called *regular functions*, of a quaternionic variable. F. Brackx[1]-[2] also developped the theory of quaternionic functions, called *monogenic functions*, of a quaternionic variable in the view point of (1).

By the second view point, an element z of the field $\mathcal{H}$ of quaternions is regarded as

$$z = x_1 + P, \qquad P = ix_2 + jx_3 + kx_4, \qquad x_1, x_2, x_3, x_4 \in \mathbf{R}. \tag{3}$$

In the fashion of (3), C. A. Deavours[4] developped the theory of quaternionic regular functions.

As the third view point, we associate two complex numbers

$$z_1 := x_1 + ix_2, \quad z_2 := x_3 + ix_4 \in \mathbf{C} \tag{4}$$

to (1), regarding as

$$z = z_1 + z_2 j \in \mathcal{H}. \tag{5}$$

Thus, we identify $\mathcal{H}$ with $\mathbf{C}^2 \cong \mathbf{R}^4$ and speak of the topology of $\mathcal{H}$. We define the non-commutative multiplication of two quaternions $z = z_1 + z_2 j, w = w_1 + w_2 j \in \mathcal{H}$ by

$$zw := (z_1 w_1 - z_2 \overline{w_2}) + (z_1 w_2 + z_2 \overline{w_1}) j \in \mathcal{H} \tag{6}$$