Nihonkai Math. J.
Vol.6 (1995), 25-34

On G-vector bundles with bracket operations and an algebra with universal mapping property

Dedicated to Professor Tsuyoshi Watabe on his 60-th birthday

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§0. Introduction.

In this paper we shall consider a class of algebras of G-invariant smooth sections of G-vector bundles over G-manifolds with bracket operations for a compact Lie group G. The class contains the Lie algebras of G-invariant smooth vector fields on G-manifolds.

Let W be a Riemannian manifold and G be a compact Lie subgroup of isometries of W. For an integer n, we shall construct an algebra $\Gamma_n^G(W)$ with a bracket operation which has a universal mapping property. The purpose of this paper is to investigate the geometric properties of the algebra $\Gamma_n^G(W)$.

Let M be an m-dimensional G-submanifold of W and h be a G-invariant smooth section of $G_n(W)|M$, where $G_n(W)$ is the bundle of n-planes over W. Then h defines a smooth G-bundle ξ_h over M and a bracket structure of the set of G-invariant smooth sections $\Gamma^G(M,h)$ of ξ_h and induces an epimorphism $\hat{\mu}(h) : \Gamma_n^G(W) \to \Gamma^G(M,h)$. We shall determine the condition that $\hat{\mu}(h)$ is bracket preserving (see Theorem 2.2). Also, by using $\hat{\mu}(h)$, we shall describe the conditions for a G-vector bundle ξ_h to be G-involutive and to be integrable in the case that G is a finite group (see Theorem 2.3 and Corollary 2.4).

Especially if $h_M : M \to \Gamma_m^G(W)$ is a map associated to the tangent space $\tau(M)$ of M, then $\hat{\mu}(h_M)$ is a bracket preserving epimorphism from $\Gamma_m^G(W)$ to the Lie algebra $\mathfrak{X}_G(M)$ of G-invariant smooth vector fields on M. In the