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# Generalized Metrics For Second Order Equations Satisfying Huygens' Principle

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**1. Introduction.** Let $M$ be an $n$-dimensional manifold without boundary of class $C^\infty$ and $\pi : TM \to M$ the tangent bundle of $M$. A second order equation on $M$ such that it is locally expressed by

$$\frac{d^2x^i}{dt^2} = F^i\left(x^1, \cdots, x^n, \frac{dx^1}{dt}, \cdots, \frac{dx^n}{dt}\right) \tag{1.1}$$

is considered to be a vector field $V$ on $TM$ with $\pi_* V(y) = y$, where $(U; x^1, \cdots, x^n)$ and $(TU; x^1, \cdots, x^n, y^1, \cdots, y^n)$ are local coordinate neighborhoods in $TM$, respectively. Restricting its domain to a hypersurface $S$ in $TM$, we define second order equations on $S$ satisfying the Huygens principle as follows ([4]). Let $f^t : TM \to TM$ be the local one-parameter group of diffeomorphisms generated by $V$. We assume that there exists a hypersurface $0 \notin S$ in $TM$ such that $S$ is $f^t$-invariant and each fibre $S_p$, $p \in M$, is a hypersurface in $T_pM$. We say that the local one-parameter group of diffeomorphisms $f^t : S \to S$ satisfies the Huygens principle if there exists a complementary $f^t$-invariant distribution $D$ on $S$, where $D$ is by definition such that

(1) $\dim D = \dim S - 1 = 2n - 2$,
(2) $V(y) \notin D(y)$ for any $y \in S$,
(3) $D(y) \supset T_yS_q$, $q = \pi(y)$, for any $y \in S$,
(4) $f^t{}_* D(y) = D(f^t y)$ for any $y \in S$, where $f^t{}_*$ is the differential map of $f^t$.

We proved in [4] that $D$ is the natural almost contact structure of $S$. Further, we showed some conditions equivalent to the principle. In particular, we were suggested to use methods developed in Riemannian geometry for the investigation of second order equations satisfying the Huygens principle. The purpose of the present paper is to introduce a generalized metric and the connection of Rund type, and try to find out what condition on this connection allows us to use them in the same way as in Riemannian geometry.

In Section 2 we need to study the relation between the second order equation on $S$ satisfying the Huygens principle and the equation of extremals of variational principle $\int L(x, \dot{x})\, dt$ in order to define a generalized