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SOME PROPERTIES OF CERTAIN ANALYTIC FUNCTIONS

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1. Introduction

Let $A(p)$ denote the class of functions of the form

$$f(z)=z^p+\sum_{k=1}^{\infty}a_{p+k}z^{p+k} \quad (p=1,2,\cdots) \tag{1}$$

which are analytic in the unit disc $D=\{z:|z|<1\}$, and $A(1)=A$. Further, we define a function $F_\lambda(z)$ by

$$F_\lambda(z)=(1-\lambda)f(z)+\lambda zf'(z) \tag{2}$$

for $\lambda>0$ and $f(z)\in A(p)$. For

$$f(z)=\sum_{n=0}^{\infty}a_n z^n \quad \text{and} \quad g(z)=\sum_{n=0}^{\infty}b_n z^n \ ,$$

we define the Hadamard product (or convolution) by

$$f*g(z)=\sum_{n=0}^{\infty}a_n b_n z^n . \tag{3}$$

Let

$$\phi(a,c;z)=\sum_{n=0}^{\infty}\frac{(a)_n}{(c)_n}z^{n+1} \qquad z\in D,\ c\neq 0,-1,-2,\cdots ,$$

$$L(a,c)f=\phi(a,c)*f(z) \qquad f(z)\in A , \tag{4}$$

where $(\lambda)_n=\Gamma(n+\lambda)/\Gamma(\lambda)$. It is known by [1] that $L(a,c)$ maps A into itself, and if $c>a>0$, $L(a,c)$ has the integral representation

$$L(a,c)f(z)=\int_0^1 u^{-1}f(uz)d\mu(a,c-a)(u) , \tag{5}$$

where μ is the beta distribution

$$d\mu(a,c-a)(u)=\frac{u^{a-1}(1-u)^{c-a-1}}{B(a,c-a)}du .$$