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THE FIRST EIGENVALUE $\lambda_{1,p}$ OF THE p-LAPLACE OPERATOR

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ABSTRACT. In this paper, we give an estimate of the first eigenvalue $\lambda_{1,p}$ of the p-Laplace operator associated to a Riemannian manifold M^m. Precisely, we show that for $p \geq 2$

$$\lambda_{1,p} \geq \left(\frac{(m-1)k}{p-1-\frac{1}{(p-2+\sqrt{m})^2}} \right)^{p/2}$$

provided that the Ricci curvature of M is no less than $(m-1)k$ where k is a positive constant. The estimate improves a recent result by A.M.Matei and is equal to the optimal result when $p=2$.

1. INTRODUCTION AND THE STATE OF THE RESULT

Let (M, g) be an m-dimensional connected compact Riemannian manifold without boundary. The first eigenvalue of the Laplace-Beltrami operator on M has been extensively studied in mathematical literature. Many connections between this invariant and other geometrical quantities have been pointed out. Recently, there has been an increasing interest for the $p-$Laplacian operator Δ_p defined by

$$\Delta_p f := -\text{div}(|df|^{p-2}df), \qquad p > 1.$$

See [1]-[8],[10]-[12]. An eigenfunction of Δ_p is a nonzero function f such that there exists a real number λ satisfying

$$\Delta_p f = \lambda |f|^{p-2} f.$$

The real number λ is then called an eigenvalue of Δ_p on M. Obviously, 0 is an eigenvalue associated with the constant eigenfunctions. The set $\sigma_p(M)$ of the remaining eigenvalues is a nonempty, unbounded subset of $(0, \infty)$ [5]. Its infimum $\lambda_{1,p}(M) = inf\sigma_p(M)$ itself is a positive eigenvalue and we have the following variational characterization [14]

$$\lambda_{1,p}(M) = inf \left\{ \frac{\int |df|^p}{\int |f|^p}; \quad 0 \not\equiv f \in W^{1,p}(M), \quad \int |f|^{p-2} f = 0 \right\}, \tag{1.1}$$

where, and throughout this paper, the integration is over M with the standard volume element induced by the Riemannian metric. So finding first nonzero eigenvalue is related to the problem of finding the best constant in the inequality

$$|f|_{L^p} \leq C |df|_{L^p}$$

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