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ON THE GEOMETRY OF QUASI-KÄHLER MANIFOLDS WITH NORDEN METRIC

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ABSTRACT. The basic class of the non-integrable almost complex manifolds with Norden metric is considered. Its curvature properties are studied. The isotropic Kähler type of investigated manifolds is introduced and characterized geometrically.

The generalized B-manifolds are introduced in [1]. They are also known as almost complex manifolds with Norden metric in [2] and as almost complex manifolds with B-metric in [3]. In the present paper these manifolds are called almost complex manifolds with Norden metric.

The aim of the present work is to further study of the geometry of one of the basic classes of almost complex manifolds with Norden metric. This is the class of the quasi-Kähler manifolds with Norden metric, which is the only basic class with non-integrable almost complex structure.

In §1 we recall the notions of the almost complex manifolds with Norden metric, we give some of their curvature properties and introduce isotropic Kähler type of the considered manifolds.

In §2 we specialize some curvature properties for the quasi-Kähler manifolds with Norden metric and the corresponding invariants.

1. ALMOST COMPLEX MANIFOLDS WITH NORDEN METRIC

Let (M, J, g) be a $2n$-dimensional almost complex manifold with Norden metric, i.e. J is an almost complex structure and g is a metric on M such that

$$J^2X = -X, \qquad g(JX, JY) = -g(X,Y) \tag{1.1}$$

for all differentiable vector fields X, Y on M, i.e. $X, Y \in \mathfrak{X}(M)$.

The associated metric $\tilde{g}$ of g on M given by $\tilde{g}(X,Y) = g(X, JY)$ for all $X, Y \in \mathfrak{X}(M)$ is a Norden metric, too. Both metrics are necessarily of signature (n, n). The manifold $(M, J, \tilde{g})$ is an almost complex manifold with Norden metric, too.

Further, X, Y, Z, U (x, y, z, u, respectively) will stand for arbitrary differentiable vector fields on M (vectors in T_pM, $p \in M$, respectively).

The Levi-Civita connection of g is denoted by ∇. The tensor filed F of type $(0,3)$ on M is defined by

$$F(X,Y,Z) = g\big((\nabla_X J)Y, Z\big). \tag{1.2}$$

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