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Notes on minimal normal compactifications of $\mathbf{C}^2/G$

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0 Introduction

Throughout the present article, we work over the field of complex numbers.

Definition 0.1 Let S be a normal affine surface and let (X, C) be a pair of a normal compact analytic surface X and a compact (analytic) curve C on X.

(1) We call the pair (X, C) a *minimal normal compactification* of S if the following conditions are satisfied:

(i) X is smooth along C.

(ii) Any singular point of C is an ordinary double point.

(iii) $X \setminus C$ is biholomorphic to S.

(iv) For any (-1)-curve $E \subset C$, we have $(E \cdot C - E) \geq 3$.

(2) Assume that (X, C) is a minimal normal compactification of S. Then (X, C) is said to be *algebraic* if X is algebraic, C is an algebraic subvariety of X and $X \setminus C$ is isomorphic to S as an algebraic variety.

For some smooth affine surfaces, their minimal normal compacitifications have been studied by several authors. In [10], Morrow gave a list of all minimal normal compactifications of the complex affine plane $\mathbf{C}^2$ by using a result of Ramanujam [12]. Ueda [14] and Suzuki [13] studied compactifications of $\mathbf{C} \times \mathbf{C}^*$ and $(\mathbf{C}^*)^2$, where $\mathbf{C}^* = \mathbf{C} \setminus \{0\}$. In particular, Suzuki [13] gave a list of all minimal normal compactifications of $\mathbf{C} \times \mathbf{C}^*$ and $(\mathbf{C}^*)^2$.

Recently, Abe, Furushima and Yamasaki [1] studied minimal normal compactifications of $S = \mathbf{C}^2/G$, where G is a small non-trivial finite subgroup of $GL(2, \mathbf{C})$, by using the theory of the cluster sets of holomorphic mappings due to Nishino and Suzuki [11]. They gave a rough classification of the weighted dual graphs of the boundary divisors of the minimal normal compactifications of S. In most cases, the singularity type of the unique singular point of S determines the weighted dual graph of the boundary divisor. However, in the case where the singular point of S