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Isomorphism classes of quasiperiodic tilings by the projection method

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ABSTRACT. Let $\mathcal{T}(W_0)$ be the space of quasiperiodic tilings by the projection method in terms of $\mathbf{R}^d = E \oplus E^\perp$ with a lattice L and the orthogonal projection $\pi : \mathbf{R}^d \to E$. We will consider the case that $L = \mathbf{Z}^d$ or (E, L) which corresponds to an exceptional folding of Coxeter groups. We determine when two tilings in $\mathcal{T}(W_0)$ belong to the same isomorphism class if $\pi|L$ is injective. As its application we have uncountably many isomorphism classes of quasiperiodic tilings by the projection method.

1. Introduction

First, we will prepare several basic definitions. A tiling T of the space $\mathbf{R}^p$ is a countable family of closed sets called tiles: $T = \{T_1, T_2, \dots\}$ such that $\cup_{i=1}^{\infty} T_i = \mathbf{R}^p$ and $\text{Int } T_i \cap \text{Int } T_j = \phi$ if $i \neq j$. An isomorphism of tilings is bijection between families of tiles that is induced by isometry of the space $\mathbf{R}^p$. An aperiodic tiling is one that admits no translation isomorphisms to itself. A tiling satisfies the local isomorphism property if for each bounded patch of the tiling there exists a positive real number r such that a translation of its patch appears in any ball of radius r. An quasiperiodic tiling is defined to be an aperiodic tiling with the local isomorphism property.

In 1981 de Bruijn [2], [3] introduced the projection method to construct quasiperiodic tilings such as Penrose tilings. The projection method was extended to the higher dimensional hypercubic lattices [5] and to more general lattices [6]. To construct tilings by the projection method, the hypercubic lattices are most frequently used. Furthermore some famous tilings are obtained from root lattices (cf. [1]). We recall the definitions of tilings by the projection method (cf. [5],[6],[9],[12]). Let L be a lattice in $\mathbf{R}^d$. Let E be a p-dimensional subspace of $\mathbf{R}^d$, and $E^\perp$ its orthogonal complement with respect to the standard inner product. Let $\pi : \mathbf{R}^d \to E$ be the orthogonal projection