

# Einstein H-umbilical submanifolds with parallel mean curvatures in complex space forms

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## Abstract

In this paper we determine H-umbilical Einstein submanifolds with parallel mean curvatures in complex space forms with non-negative holomorphic sectional curvatures.

## 1 Introduction

In Riemannian Geometry, Einstein manifolds are very important subject. When we focus our attention to submanifolds in complex space forms, there are many interesting results (cf. [1]). There are two important classes of submanifolds of a complex space form. One is the class of holomorphic submanifolds and another is the class of totally real submanifolds. A submanifold in a complex space form is said to be totally real if the complex structure of the ambient space carries each tangent vector to a normal vector. A totally real submanifold is called a Lagrangian submanifold if its real dimension is equal to the complex dimension of the ambient space. The classification of Lagrangian Einstein submanifolds of a complex space form is still open. We know the fact that a non-flat complex space form of complex dimension  $\geq 2$  admits no totally umbilical Lagrangian submanifolds except the totally geodesic ones. So, B. Y. Chen [3] introduced the notion of H-umbilical submanifolds which are the simplest Lagrangian submanifolds next to the totally geodesic ones in a complex space form (for the definition see §2).

In this paper we investigate H-umbilical Einstein submanifolds of complex space forms with non-negative holomorphic sectional curvatures and give the following theorem:

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