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CONVERGENCE THEOREMS TO COMMON FIXED POINTS FOR INFINITE FAMILIES OF NONEXPANSIVE MAPPINGS IN STRICTLY CONVEX BANACH SPACES

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ABSTRACT. In this paper, we prove convergence theorems to common fixed points for infinite families of nonexpansive mappings in strictly convex Banach spaces. One of our results is the following: Let C be a compact convex subset of a strictly convex Banach space E. Let $\{T_n : n \in \mathbb{N}\}$ be a sequence of nonexpansive mappings on C with $\bigcap_{n=1}^\infty F(T_n) \neq \varnothing$. Let $\{\lambda_n\}$ be a sequence of positive numbers such that $\sum_{n=1}^\infty \lambda_n < 1$. Define a sequence $\{x_n\}$ in C by $x_1 \in C$ and

$$x_{n+1} = \left(1 - \sum_{i=1}^n \lambda_i\right) x_n + \sum_{i=1}^n \lambda_i T_i x_n$$

for $n \in \mathbb{N}$. Then $\{x_n\}$ converges strongly to a common fixed point of $\{T_n : n \in \mathbb{N}\}$.

1. INTRODUCTION

A mapping T on a closed convex subset C of a Banach space E is called a nonexpansive mapping if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. We denote by $F(T)$ the set of fixed points of T. We know that $F(T)$ is nonempty if E is uniformly convex and C is bounded; see Browder [1], Göhde [7], and Kirk [11]. In 1953, Mann [14] considered the following iteration scheme: $x_1 \in C$ and

$$x_{n+1} = \alpha_n T x_n + (1 - \alpha_n) x_n \tag{1}$$

for $n \in \mathbb{N}$, where $\{\alpha_n\}$ is a sequence in $[0, 1]$. Later several authors have studied Mann's iteration process; see Edelstein and O'brien [5], Groetsch [8], Ishikawa [9], Opial [15], Outlaw [16], Reich [17] and so on. For example, Reich [17] proved the following: (1) converges weakly to a fixed point z of T if E is uniformly convex and the norm of E is Fréchet differentiable, C is closed and convex, T is nonexpansive and has a fixed point, and $\{\alpha_n\}$ satisfies $\sum_{n=1}^\infty \alpha_n(1-\alpha_n) = \infty$. Also Ishikawa [9] proved the following: (1) converges strongly to a fixed point z of T if C is compact and convex, T is nonexpansive, and $\{\alpha_n\}$ satisfies $\sum_{n=1}^\infty \alpha_n = \infty$ and $\limsup_n \alpha_n < 1$. Convergence theorems for families of nonexpansive mappings are proved in Crombez [4], Ishikawa [10], Kitahara and Takahashi [12], Linhart [13], Takahashi and Tamura [21] and so on. For example, Linhart [13] proved the following:

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