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THE INDEX FORM OF A GEODESIC ON A GLUED RIEMANNIAN SPACE

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Abstract

A topological space obtained from Riemannian manifolds by identifying their isometric submanifolds is called a glued Riemannian space. In this space, we consider the variational problem with respect to arc length L of piecewise smooth curves through the identified submanifold B. The first variation formula shows that a critical point of L is a curve which is a geodesic on each Riemannian manifold and satisfies certain passage law on B. We call this curve a B-geodesic. The second variation formula for a B-geodesic is also obtained. Moreover, we study the index form and B-conjugate points for a B-geodesic in this variational problem. Especially, in a glued Riemannian space constructed from Riemannian manifolds of constant curvature, we have the passage equation which make the relation between the shape operator and the first B-conjugacy clear.

0. Introduction

In Riemannian manifolds, various results have been given on geodesics by many authors. Recently, N.Innami studied a geodesic reflecting at a boundary point of a Riemannian manifold with boundary in [4]. Let M be a Riemannian manifold with boundary B which is a union of smooth hypersurfaces. A curve on M is said to be a reflecting geodesic if it is a geodesic except at reflecting points and satisfies the reflection law. He dealt with the index form, conjugate points and so on, as in the case of a usual geodesic. Moreover, in [5], he generalized these to the case of a glued Riemannian manifold which is a space obtained from Riemannian manifolds with boundary by identifying their isometric boundary hypersurfaces.

The purpose of this paper is to generalize some of his results to the case of a glued Riemannian space, which is obtained from Riemannian manifolds M_1 and M_2 (we allow the case where $\dim M_1 \neq \dim M_2$) by identifying their isometric submanifolds B_1 and B_2. The detailed definition will be described in Section 1. We consider the variational problem with respect to arc length L of piecewise smooth curves through the identified submanifold B. The first variation formula shows that a critical point of L is a curve which is a geodesic