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ESTIMATING COMMON FIXED POINTS OF TWO NONEXPANSIVE MAPPINGS BY STRONG CONVERGENCE

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ABSTRACT. In this paper, we introduce an iteration scheme defined by

$$x_0 = x \in C, \ x_{n+1} = \alpha_n x + (1-\alpha_n)U_n x_n,$$
$$U_n = \gamma_n T(\beta_n S + (1-\beta_n)I) + (1-\gamma_n)I, \ n = 0, 1, 2, \ldots,$$

where S and T are nonexpansive mappings from a closed convex subset of a Banach space into itself and $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are in [0,1]. This scheme contains all four schemes given by Mann, Ishikawa, Das–Debata and Halpern. Using this scheme we approximate common fixed points of S and T.

1. INTRODUCTION

Let C be a closed convex subset of a Banach space E. A mapping $T : C \to C$ is called nonexpansive if $\|Tx - Ty\| \le \|x - y\|$ for all $x, y \in C$. A number of iteration schemes have been introduced to approximate the fixed points of nonexpansive mappings. Mann [6] introduced the following iteration scheme :

$$(1) \qquad x_1 \in C, \ x_{n+1} = \alpha_n T x_n + (1-\alpha_n)x_n,$$

for all $n = 1, 2, \ldots$, where $\{\alpha_n\}$ is in $[0, 1]$. Ishikawa [4] gave the following iteration scheme:

$$(2) \qquad x_1 \in C, \ x_{n+1} = \alpha_n T(\beta_n T x_n + (1-\beta_n)x_n) + (1-\alpha_n)x_n,$$

for all $n = 1, 2, \ldots$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are in $[0, 1]$. Das and Debata [2] defined $\{x_n\}$ using two mappings S and T as follows:

$$(3) \qquad x_1 \in C, \ x_{n+1} = \alpha_n S(\beta_n T x_n + (1-\beta_n)x_n) + (1-\alpha_n)x_n$$

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