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On an Invariant Subspace Whose Common Zero Set is the Zeros of Some Function

By

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Abstract. Let F be a nonzero function in $H^2(D^n)$ such that if ϕ is a function in $L^\infty(T^n)$ and ϕF is in $H^2(D^n)$, then ϕ belongs to $H^\infty(D^n)$. We study the set of multipliers of an invariant subspace M of $H^2(D^n)$ whose common zero set of M is just a zero set of F.

§1. Introduction

Let D^n be the open unit polydisc in $\mathbf{C}^n$ and T^n be its distinguished boundary. The normalized Lebesgue measure on T^n is denoted by dm. For $0 < p \leq \infty$, $H^p(D^n)$ is the Hardy space and $L^p(T^n)$ is the Lebesgue space on T^n. Let $N(D^n)$ denote the Nevanlinna class. Each f in $N(D^n)$ has radial limits f^* defined on T^n *a.e.*. Moreover, there is a singular measure $d\sigma_f$ on T^n determined by f such that the least harmonic majorant $u(\log|f|)$ of $\log|f|$ is given by $u(\log|f|)(z) = P_z(\log|f^*| + d\sigma_f)$ where P_z denotes Poisson integration and $z = (z_1, z_2, \cdots, z_n) \in D^n$. Put $N_*(D^n) = \{f \in N(D^n) \ ; \ d\sigma_f \leq 0\}$, then $H^p(D^n) \subset N_*(D^n) \subset N(D^n)$ and $H^p(D^n) = N_*(D^n) \cap L^p(T^n) \subsetneq N(D^n) \cap L^p(T^n)$. These facts are shown in [10, Theorem 3.3.5].

A closed subspace M of $H^p(D^n)$ is said to be invariant if $z_jM \subset M$ for $j = 1, 2, \cdots, n$. For an invariant subspace M of $H^2(D^n)$, set

$$\mathcal{M}(M) = \{\phi \in L^\infty(T^n) \ ; \ \phi M \subseteq H^2(D^n)\}.$$

$\mathcal{M}(M)$ is called the set of multipliers of M and $\mathcal{M}(M) \supseteq H^\infty(D^n)$. $\mathcal{M}(M)$ has been studied in [1],[2],[3],[7],[8] and [9]. In the previous paper [7], the author studied $\mathcal{M}(M)$ in general and gave a necessary and sufficient condition for $\mathcal{M}(M) = H^\infty(D^n)$. It is easy to see that $\mathcal{M}(M) = H^\infty(D^n)$ when the condimension of M in $H^2(D^n)$ is finite.