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ON ALGEBRAICALLY TOTAL $*$–PARANORMALITY

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ABSTRACT. In this paper, we introduce the notion of algebraically $*$-TPN operators on a Hilbert space H as : An operator T is algebraically $*$-TPN if there exists a nonconstant complex polynomial p such that $p(T)$ is totally $*$-paranormal. In particular, we prove that this class of $*$-TPN (or equivalently, totally $*$-paranormal) operators forms a proper subclass of algebraically $*$-TPN operators. Also we prove that Weyl's theorem and the spectral mapping theorem hold for algebraically $* - TPN$ operators. Finally, we prove that if T is algebraically $*$-TPN, then $f(T)$ satisfies Weyl's theorem where f is analytic on an open neighborhood of $\sigma(T)$.

0. Introduction

Let H be an infinite dimensional complex Hilbert space and $\mathcal{L}(H)$ denote the space of all bounded linear operators from H to H. If $T \in \mathcal{L}(H)$, we write $N(T)$ and $R(T)$ for the null space and range of T; $\sigma(T)$ for the spectrum of T and $\sigma_e(T)$ for the essential spectrum of T. Recall that an operator $T \in L(H)$ is *Fredholm* if its range $R(T)$ is closed and both the null spaces $N(T)$ and $N(T^*)$ are finite dimensional. The *index* of a Fredholm operator T, denoted by ind(T), is defined by

$$\text{ind}(T) = \dim\, N(T) - \dim\, N(T^*)(= \dim\, N(T) - \dim\, R(T)^{\perp}).$$

An operator $T \in \mathcal{L}(H)$ is called Weyl if T is a Fredholm operator of index zero. The *Weyl spectrum* of T, denoted by $\omega(T)$, is defined by the formula

$$\omega(T) = \{\lambda \in \mathbb{C} :\ T - \lambda I \text{ is not Weyl}\}.$$

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