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EXISTENCE OF NONEXPANSIVE RETRACTIONS AND MEAN ERGODIC THEOREMS IN HILBERT SPACES

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Abstract

Let C be a nonempty closed convex subset of a Hilbert space H. Let S be a semigroup and let $\mathcal{S} = \{T_t : t \in S\}$ be an asymptotically nonexpansive semigroup on C such that the set $F(\mathcal{S})$ of common fixed points of $\mathcal{S}$ is nonempty. We consider the existence of an ergodic retraction and prove that if $\{\mu_\alpha\}$ is an asymptotically invariant net of means, then for each $x \in C$, $\{T_{\mu_\alpha} x\}$ converges weakly to an element of $F(\mathcal{S})$.

1 Introduction

Let C be a nonempty closed convex subset of a real Hilbert space H. Then, a mapping $T : C \to C$ is said to be *Lipschitzian* if there exists a nonnegative real number k such that

$$\|Tx - Ty\| \leq k\|x - y\| \quad \text{for every} \quad x, y \in C.$$

T is said to be *nonexpansive* if $k = 1$. Let S be a semigroup. Then, a family $\mathcal{S} = \{T_t : t \in S\}$ of mappings from C into itself is said to be a *Lipschitzian semigroup* on C with Lipschitz constants $\{k_t : t \in S\}$ if it satisfies the following:

(1) for each $t \in S$, there exists a nonnegative real number k_t such that

$$\|T_t x - T_t y\| \leq k_t\|x - y\| \quad \text{for every} \quad x, y \in C;$$

(2) $T_{st}x = T_s T_t x$ for every $s, t \in S$ and $x \in C$.

We denote by $F(\mathcal{S})$ the set of common fixed points of $\mathcal{S}$. $\mathcal{S}$ is said to be a *nonexpansive semigroup* on C if $k_t = 1$ for every $t \in S$. $\mathcal{S}$ is also said to be an *asymptotically nonexpansive semigroup* on C if $\inf_s \sup_t k_{ts} \leq 1$ and $\sup_t k_t < \infty$. In particular, $\mathcal{S}$ is said to be a *one-parameter asymptotically nonexpansive semigroup* on C if $S = [0, \infty)$ and for each $x \in C$, the mapping $t \mapsto T_t x$ from S into C is continuous.

The first nonlinear ergodic theorem for nonexpansive mappings was established in 1975 by Baillon [1]: Let C be a closed convex subset of a Hilbert space and let T be a