Some hypersurfaces in a Euclidean space

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1. Introduction.

The Riemannian curvature tensor R of a locally symmetric Riemannian manifold (M, g) satisfies

$$(*) \qquad R(X, Y)\cdot R=0, \qquad \text{for any tangent vectors } X \text{ and } Y,$$

where the endomorphism $R(X, Y)$ operates on R as a derivation of the tensor algebra at each point of M. A result of K. Nomizu [2] tells us that the converse is affirmative in the case where M is a certain hypersurface in a Euclidean space. That is

THEOREM A. *Let M be an m-dimensional, connected and complete Rimannian manifold which is isometrically immersed in a Euclidean space E^{m+1} so that the type number $k(x)\geqq 3$ at least at one point x. If M satisfies the condition $(*)$, then it is of the form $M=S^k\times E^{m-k}$, where S^k is a hypersphere in a Euclidean subspace E^{k+1} of E^{m+1} and E^{m-k} is a Euclidean subspace orthogonal to E^{k+1}.*

Now, let R_1 be the Ricci tensor of M and R^1 be the symmetric endomorphism given by $R_1(X, Y)=g(R^1X, Y)$. Then, the condition $(*)$ implies in particular

$$(**) \qquad R(X, Y)\cdot R_1=0, \qquad \text{for any tangent vectors } X \text{ and } Y.$$

Recently, S. Tanno [4] gave the following

THEOLEM B. *Let M be an m-dimensional, connected and complete Rimannian manifold which is isometrically immersed in a Euclidean space E^{m+1} so that the type number $k(x)\geqq 3$ at least at one point x. If M satisfies the condition $(**)$ and have the positive scalar curvature, then it is of the form $M=S^k\times E^{m-k}$.*

In the present paper, we shall show that the assumption of having the positive scalar curvature in theorem B can be replaced by some other conditions. That is :

THEOREM C. *Let M be an m-dimensional, connected and complete Riemannian manifold which isometrically immered in a Euclidean space E^{m+1} so that M is not minimal and the type number $k(x)\geqq 3$ at least at one point x. If M satisfies the condition $(**)$, then it is of*