C*-algebras having the property (T)

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1. Introduction

If A and B are C*-algebras, $A\odot B$ denotes their algebraic tensor product. A norm $\|\ \|_\beta$ in $A\odot B$ is called *compatible* if the completion of $A\odot B$ by $\|\ \|_\beta$ becomes a C*-algebra, and we denote by $A\otimes_\beta B$ the C*-algebra which is the completion of $A\odot B$ with respect to $\|\ \|_\beta$. There are some ways to define compatible norms in $A\odot B$. T. Turumaru [5] introduced the α-norm. As A. Wulfsohn established, the α-norm has the property:

$$\|\sum_{k=1}^{n} x_k\otimes y_k\|_\alpha=\|\sum_{k=1}^{n}\pi_1(x_k)\otimes\pi_2(y_k)\|,\quad x_k\in A,\ y_k\in B$$

where π_1 and π_2 are any faithful representations of A and B, respectively. M. Takesaki proved in [4] that the α-norm is not necessarily the unique compatible norm in $A\odot B$ and that it is the least one among the all compatible norms.

On the other hand, A. Guichardet defined the ν-norm and showed that it is the greatest one among the all compatible norms. The ν-norm is defined by the formula

$$\|x\|_\nu=\sup_\pi \|\pi(x)\|,\quad x\in A\odot B$$

where π runs over the set of all representations of $A\odot B$ which are continuous with respect to any compatible norm in $A\odot B$.

We say that a C*-algebra A has *the property* (T) if, for every C*-algebra B, the α-norm in $A\odot B$ is the unique compatible norm.

This papar is concerned with C*-algebras having the property (T). In § 2, we consider the structure of C*-algebras having the property (T). In §3, we apply the consideration in §2 to tensor products of C*-algebras. Finally in §4 we present that a C*-algebra A has the greatest closed two-sided ideal I having the property (T) and it is the least one such that A/I has no nonzero closed two-sided ideals having the property (T).