On imbedding closed 4-dimensional manifolds in Euclidean space

By

Tsuyoshi WATABE

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1. Introduction

It is an open question to give an imbedding of an oriented closed differentiable 4-manifold in 7-dimensional Euclidean space space R^7 [1]. Recently, M. Hirsch has proved that such a manifold can be imbedded in R^7 piecewise linearly [4]. A closed n-dimensionaly manifold M^n will be said to be *almost differentiably imbeddable* in R^m if $M^n - x$, where x is a point of M, is differentiably imbeddable in R^m. It is known that a closed differentiable 4-manifold is almost differentiably imbeddable in R^7 [3].

In what follows, all manifolds are understood to be differentiable and compact. Differentiable will always mean of class C^∞. The notation R^n will be used for the n-dimensional Euclidean space. We write $M_1 \approx M_2$ if M_1 and M_2 are diffeomorphic The notation $\#$ will mean of the connected sum defined in [7].

In this paper, we shall prove the following

THEOREM 1. *All 4-dimensional closed π-manifolds are imbeddable in R^7.*

THEOREM 2. *All simply connected closed 4-dimensional π-manifolds are imbeddable in R^6.*

THEOREM 3. *All homotopy 4-spheres are imbeddable in R^5.*

The result of Theorem 3 has been obtained by S. Smale (unpublished).

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2. Imbedding of homotopy spheres

It is known that all homotopy n-spheres are imbeddable in R^{n+k}, where $n<2k-2$ [2]. It is easy to show that a homotopy n-sphere is imbeddable in R^{n+1} if and only if it is h-cobordant to the standard n-sphere S^n. Hence if n is greater than 4 the standard n-sphere is the only homotopy n-sphere which is imbeddable in R^{n+1}.

According to a result of [3], we can prove the following (we assume $n>5$)