ON THE LEARNING ALGORITHM OF 2-PERSON ZERO-SUM GAME

By

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1. Introduction

In view of practical problems, the methods of learning a pair of optimal strategies, which is called a solution of game, have been investigated by many authors (see, for example, [1], [2], [3] and [4]). Especially, a type of the learning algorithm in [4] is pseudogradient one which uses the idea of regularization for supplying the lack of the strict convexity of the payoff function.

But their algorithm does not seem general enough to us in point of utilizing the given information. For this reason, we propose a learning algorithm which is an extension of their algorithm. Then, we show that a pair of the mixed strategies generated by our algorithm converges with probability one (w. p. 1) and in mean square to a pair of the optimal mixed strategies under some conditions. Moreover, we give an upper bound for the rate of convergence of our algorithm.

2. Formulation of 2-person zero-sum game

In this section, we consider a zero-sum game with two players (player I and player II) consisting of an infinite number of repeated single games under an assumption of incomplete information relating to payoff matrix. Player I and player II have, respectively, m and l available pure strategies. If player I has chosen a pure strategy with the number i and player II has chosen one with the number j, then their payoffs consist of ξ and η, respectively, which are random variables such that

$$\mathrm{E}[\xi \mid i, j] = -\mathrm{E}[\eta \mid i, j] = r_{ij}, \quad |r_{ij}| < \infty, \tag{1}$$

$$\mathrm{E}[\xi^2 \mid i, j] = R_{ij}^{(1)} < \infty, \quad \mathrm{E}[\eta^2 \mid i, j] = R_{ij}^{(2)} < \infty. \tag{2}$$

In general, the payoff matrix $A=(r_{ij})$ is assumed to be known in advance. But, in this game, the payoff matrix is not known and in each single game the players receive information relating to the payoff matrix A in the form of realization of ξ and η.

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