On the degree of symmetry of complex quadric and homotopy complex projective space

By

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Introduction

Let M be a compact connected differentiable manifold of dimension $2n$. Following [8], we define $N(M)$, *the degree of symmetry of* M, the maximum of dimension of isometry groups of all possible Riemannian structures on M. Of course, $N(M)$ is the maximum of dimensions of compact connected Lie groups which can act almost effectively on M.

In this note, we shall consider the degree of complex quadric $Q_n=SO(n+2)/SO(2)\times SO(n)$ and homotopy complex projective space CP_n.

In [8], W. Y. Hsiang has proved the following

THEOREM. $N(CP_n)=\dim SU(n+1)=n^2+2n$.

We have the following

THEOREM A. *Let M be a closed differentiable manifold of dimension $2n$ which is homotopy equivalent to CP_n. Assume that $n\geqq 13$. If $N(M)\geqq(1/2)(n^2+3n+2)$, then M is diffeomorphic to CP_n.*

As a corollary of this we have the following

THEOREM B. *The degree of symmetry of an exiotic homotopy complex projective space of dimension of $2n$ is less than* $(1/2)(n^2+3n+2)$. $(n\geqq 13)$

For a complex quadric Q_n, we have the following

THEOREM C. $N(Q_n)=\dim SO(n+2)=(1/2)(n^2+3n+2)(n\geqq 13)$.

In section 1, we state the results and prove Theorem A and C modulo lemmas and propositions which are proved in later sections.

In this note all actions are differentiable.

1. Statement of results

A closed differentiable manifold M^{2n} is said to be *homologically kählerian* if there exists an element $a\in H^2(M; Q)$ (Q=rationals) such that the multiplication by a^{n-s}($s=0,$