On the asymptotic distribution of eigenvalues of non-symmetric operators associated with strongly elliptic sesquilinear forms

By

Michiaki WATANABE

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1. Introduction

The asymptotic behaviour of eigenvalues of elliptic operators has been investigated by a number of authors. Combining an asymptotic expansion theorem for resolvent kernels, established by S. Agmon and Y. Kannai [3], and Malliavin's tauberian theorem together with the formula of Å. Pleijel [8], S. Agmon [2] deduced asymptotic formulas with remainder estimates for eigenvalues of operators whose coefficients are infinitely differentiable. Recently K. Maruo and H. Tanabe [7] have devised a method of estimating resolvent kernels of a class of operators with wider domains and improved the results of S. Agmon to obtain estimates for remainders in the asymptotic formulas under less smoothness assumptions on the coefficients of operators. And these results were extended to the case of some non-symmetric operators and the remainder estimates were strengthened further by K. Maruo [6]. We refer to the above results for our discussion.

The above authors have, however, always confined themselves to the case where operators dealt with are self-adjoint in some sense or other. From the viewpoint of pure theory at least we consider it desirable to extend the above results to more general non-symmetric cases. The purpose of this paper is to deal with operators which are not necessarily symmetric but satisfy the condition (3) stated below and to deduce asymptotic formulas for their eigenvalues slightly different from those obtained by the above authors.

Let Ω be a bounded domain in the real space R^n. We denote by $H_m(\Omega)$ for an integer $m \geqq 0$ the subclass of functions $u \in L^2(\Omega)$ with *distribution derivatives* $D^\alpha u \in L^2(\Omega)$ for all $|\alpha| \leqq m$, where and in the following $\alpha=(\alpha_1,\dots,\alpha_n)$ is a multiindex of length $|\alpha|=|\alpha_1|+\cdots+|\alpha_n|$ and $D^\alpha=D_1^{\alpha_1}\dots D_n^{\alpha_n}$, $D_k=(\partial/\partial x_k)$, $k=1,\dots,n$.

In $H_m(\Omega)$ we introduce as usual the inner product and the norm:

$$(u, v)_m=(u, v)_{m,\Omega}=\left(\int_\Omega \sum_{|\alpha|\leqq m} D^\alpha u\ \overline{D^\alpha v}\, dx\right)^{1/2};\ \|u\|_m=\|u\|_{m,\Omega}=((u, u)_m)^{1/2}.$$