Boundary representations of a tensor product of C*-algebras

By

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1. Introduction

In [1] Arveson gives a non-commutative generalization of Choquet boundary and Silow boundary. We shall study those of a tensor product of C*-algebras.

If E is a subspace of a C*-algebra and M_n is the algebra of $n \times n$ complex matrices, then the algebraic tensor product $E \otimes M_n$ is the set of $n \times n$ matrices with entries in E. If φ: $E \rightarrow F$ is a linear map from one linear space into another, then, for each positive integer n, define φ_n: $E \otimes M_n \longrightarrow F \otimes M_n$ by applying element by element to each matrix over E, i.e. $\varphi_n(T_{ij}) = (\varphi(T_{ij}))$. φ is called completely positive (resp., completely isometric) if each φ_n is positive (resp., isometric).

Following Arveson [1], let B be a C*-algebra with unit and A a subspace of B which contains unit and generates B as a C*-algebra.

An irreducible representation π of B is called a boundary representation for A if the restriction $\pi | A$ has a unique completely positive linear extension to B.

A closed two-sided ideal J in B is called a boundary ideal for A if the canonical quotient map q_J: $B \longrightarrow B/J$ is completely isometric on A.

A boundary ideal is called the Silov boundary for A if it contains every other boundary ideal.

A is called an admissible subspace of B if the intersection of the kernels of the boundary representatoins for A is a boundary ideal for A.

Throughout this paper, we use the following notations. Let B_1 and B_2 be C*-algebras, and let for each $i=1, 2$, e_i be unit in B_i, A_i a subspace of B_i which contains e_i and generates B_i as a C*-algebra.

2. Boundary representations

Let $A_1 \otimes A_2$ be the algebraic tensor product, and $B_1 \underset{\alpha}{\otimes} B_2$ the C*-tensor product [3]. Then $A_1 \otimes A_2$ generates $B_1 \underset{\alpha}{\otimes} B_2$ as a C*-algebra.