Some 3-dimensional Riemannian manifolds with constant scalar curvature

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1. Introduction

Let (M, g) be Riemannian manifold. By R we denote the Riemannian curvature tensor. By $T_x(M)$ we denote the tangent space to M at x. Let $X, Y \in T_x(M)$. Then $R(X, Y)$ operates on the tensor algebra as a derivation at each point x. In a locally symmetric space ($\nabla R=0$), we have

(*) $\quad R(X, Y) \cdot R=0$ for any point $x \in M$ and $X, Y \in T_x(M)$.

We consider the converse under some additional conditions.

THEOREM A (S. Tanno [7]). *Let (M, g) be a complete and irreducible 3-dimensional Riemannian manifold. If (M, g) satisfies (*) and the scalar curvature S is positive and bounded away from 0 on M, then (M, g) is of positive constant curvature.*

THEOREM B (K. Sekigawa [5]). *Let (M, g) be a compact and irreducible 3-dimensional Riemannian manifold of class C^∞ satisfying (*). If the rank of the Ricci tensor R_1 is non-zero on M, then (M, g) is of constant curvature.*

In this note, we shall prove the followings

THEOREM C *Let (M, g) be a compact and irreducible 3-dimensional Riemannian manifold satisfying (*). If the scalar curvature S is constant, then (M, g) is of constant curvature.*

THEOREM D *Let (M, g) be a 3-dimensional homogeneous Riemanian manifold satisfying (*). Then (M, g) is either*

(1) *a space of constant curvature, or*

(2) *a locally product Riemannian manifold of a 2-dimensional space of constant curvature and a real line.*

It may be noticed that (*) is equivalent to (**) $R(X, Y) \cdot R_1=0$. In this note, (M, g) is assumed to be connected and of class C^∞.