The second dual of a tensor product of C*-algebras, II

By

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1. Introductinn

Let C be a C*-algebra, and let π_C be the universal represensation of C in the universal representation Hilbert space H_C. The second dual C^{**} of C may be identified with the closure of $\pi_C(C)$ in weak operator topology [1: p. 236]. For C*-algebras A and B we denote by $A\underset{\alpha}{\otimes}B$ the C*-tensor product of A and B, $A^{**}\otimes B^{**}$ the W*-tensor product of A^{**} and B^{**}. Since there exists the canonical *-isomorphism $\pi_A\otimes\pi_B$ from $A\underset{\alpha}{\otimes}B$ into $A^{**}\underset{\alpha}{\otimes}B^{**}$, $A\underset{\alpha}{\otimes}B$ may be identified with the weak dense subalgebra $\pi_A\otimes\pi_B\,(A\underset{\alpha}{\otimes}B)$ of $A^{**}\otimes B^{**}$. In this paper we shall study positive linear functionals of $A\underset{\alpha}{\otimes}B$ which has the normal extension to $A^{**}\otimes B^{**}$.

In §2, we shall show a characterization of pure states having the normal extension to $A^{**}\otimes B^{**}$.

In §3, we shall show that $(A\underset{\alpha}{\otimes}B)^{**}$ is *-isomorphic to $A^{**}\otimes B^{**}$ when either A or B is a dual C*-algebra, and the *-isomorphism $\pi_A\otimes\pi_B$ has no normal extension to $(A\underset{\alpha}{\otimes}B)^{**}$ when A and B are UHF algebras [2: Definition 1. 1].

2. Theorem

THEOREM. *Let A and B be C*-algebras and π be an irreducible representation of $A\underset{\alpha}{\otimes}B$ on a Hilbert space $H\pi$. Then the following two assertions are equivalent.*

(a) π is equivalent with a representation $\pi_1\otimes\pi_2$ where π_1 and π_2 are representations of A and B, respectively.

(b) A positive linear functional f of $A\underset{\alpha}{\otimes}B$ has the normal extension to $A^{**}\otimes B^{**}$, where f is given by the formula

$$f(x)=(\pi(x)\xi,\ \xi),\quad x\in A\underset{\alpha}{\otimes}B,\quad \xi\in H_\pi.$$

PROOF. It is obvious that (a) implies (b).

If (b) holds, f can be expressed such that

$$f(x)=(x\xi,\ \xi),\quad x\in A\underset{\alpha}{\otimes}B,\quad \xi\in H_A\otimes H_B.$$