Methods and Applications of Analysis.
Vol. 16, No. 4, pp. 479–490, December 2009

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ON BREAKDOWN OF SOLUTIONS TO THE FULL COMPRESSIBLE NAVIER-STOKES EQUATIONS*

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Abstract. In this paper, when the initial density is away from vacuum, we establish a blow up criterion for the strong solutions of the viscous heat-conductive flows just in terms of the gradients of the velocity and the temperature, analogous to the Beal-Kato-Majda criterion for the ideal incompressible flow. In particular, the viscous coefficients μ and λ are only required to satisfy the physical restrictions.

Key words. Breakdown, full compressible Navier-Stokes.

AMS subject classifications. 35Q30, 76N10

1. Introduction. This paper is devoted to studying the following 3D full compressible $Navier-Stokes$ equations:

$$\begin{cases} \partial_t\rho + \mathrm{div}(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}(\rho u\otimes u) - \mu\triangle u - (\mu+\lambda)\nabla(\mathrm{div}u) + \nabla P = 0 \\ c_v[\partial_t(\rho\theta) + \mathrm{div}(\rho\theta u)] - \kappa\triangle\theta + P\mathrm{div}u = \dfrac{\mu}{2}|\nabla u + \nabla u^T|^2 + \lambda(\mathrm{div}u)^2, \end{cases} \tag{1.1}$$

where $\rho\geq 0$ denotes the density of the mass; u is the velocity;

$$P = R\rho\theta \quad (R>0) \tag{1.2}$$

is the pressure; μ,λ,R,c_v and κ are the physical constants satisfying $\mu>0, \lambda+\frac{2\mu}{3}\geq 0, R>0, c_v>0$ and $\kappa>0$.

Let Ω be a bounded smooth domain in R^3. We consider an initial boundary value problem for $(1.1)-(1.3)$ with the following boundary conditions

$$(\rho,u,\theta)|_{t=0} = (\rho_0,u_0,\theta_0) \quad \text{in} \quad \Omega \tag{1.3}$$

$$u|_{\partial\Omega} = 0,\ \frac{\partial\theta}{\partial\nu}|_{\partial\Omega} = 0 \tag{1.4}$$

where ν is the normal to $\partial\Omega$.

There are huge literatures on the studies of the well-posedness and behavior of solutions to (1.1). In the case that the density is away from vacuum, the one-dimensional problem was addressed by Kazhikhov and Shelukhin [26] for sufficient smooth data, and by Serre [33,34] and Hoff [18] for discontinuous initial data. The global existence of classical solutions to the compressible Navier-Stokes equations in multidimensional case was obtained by Matsumura and Nishida [31] as long as the initial data is a small perturbation of a non-vacuum constant state in H^3. This result was generalized to

*Received December 11, 2009; accepted for publication January 11, 2010.
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