

THE RELATIVE SYMPLECTIC CONE AND T^2 -FIBRATIONS

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In this note we introduce the notion of the relative symplectic cone \mathcal{C}_M^V . As an application, we determine the *symplectic cone* \mathcal{C}_M of certain T^2 -fibrations. In particular, for some elliptic surfaces we verify the conjecture in [17]: If M underlies a minimal Kähler surface with $p_g > 0$, the symplectic cone \mathcal{C}_M is equal to $\mathcal{P}^{c_1(M)} \cup \mathcal{P}^{-c_1(M)}$, where $\mathcal{P}^\alpha = \{e \in H^2(M; \mathbb{R}) | e \cdot e > 0 \text{ and } e \cdot \alpha > 0\}$ for nonzero $\alpha \in H^2(M; \mathbb{R})$ and $\mathcal{P}^0 = \{e \in H^2(M; \mathbb{R}) | e \cdot e > 0\}$.

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