



BOOK REVIEW

Lectures on Curves, Surfaces and Projective Varieties (A Classical View of Algebraic Geometry), by Mauro C. Beltrametti, Ettore Carletti, Dionisio Gallarati and Giacomo M. Bragadin, European Mathematical Society Textbooks, Zürich, 2009, xv + 491pp., ISBN 978-3-03719-064-7.

The book under review represents the algebraic-geometric trends from nineteenth century and the first half of twentieth century. The motivation of the authors is not only to recall the history, but to introduce the young algebraic geometers to the natural constructions and ideas, underlying the abstract contemporary treatment.

The first chapter collects some preliminaries on projective spaces and correspondences, as well as on the duality between points and projective hyperplanes.

The second one recollects the notion of Zariski topology and Hilbert's Nullstellensatz on the correspondence between affine algebraic varieties and radical polynomial ideals. It proceeds with morphisms, rational maps and Noether Normalization Lemma. Special attention is paid to the projective algebraic varieties and their associated homogeneous radical ideals.

The third chapter is devoted to the tangent space $T_p X$ and the singularities of an algebraic variety X . The dimension of an algebraic variety is defined as the infimum of the dimensions of the tangent spaces, after showing that this infimum is attained on an open Zariski dense subset. It is proved to coincide with the transcendence degree of the field of the rational functions. Meanwhile are discussed the local parameters and the intersections with projective subspaces of sufficiently large dimension. The order (or degree) of a variety X is introduced algebraically and exhibited to coincide with the number of the intersection points with a generic projective subspace of complementary dimension. As a result, there arises the notion of a tangent cone $TC_P X$ at a singular point P of X .

The fourth chapter is on the intersection multiplicity and Bezout's Theorem for coplanar algebraic curves. It exposes Kronecker's elimination of $(r - m)$ variables as a projection with center \mathbb{P}^{r-m-1} on \mathbb{P}^m .