Differential and Integral Equations, Volume 3, Number 2, March 1990, pp. 381–396.

DAMPING OPERATORS IN CONTINUUM MODELS OF FLEXIBLE STRUCTURES: EXPLICIT MODELS FOR PROPORTIONAL DAMPING IN BEAM TORSION

A.V. BALAKRISHNAN†

Laboratory for Flight Systems Research
University of California, Los Angeles, CA 90024 USA

Abstract. A new, explicit representation of damping operators for strictly proportional damping for the torsion mode of a finite beam is presented. The damping operator is the square root of the stiffness operator (enhanced to include the boundary) and is calculated using the Balakrishnan formula. It is nonlocal, and for the clamped (or "fixed") end case, turns out to be a finite-limit version of the Hilbert transform; with end mass (or boundary control), the square-root operator introduces nonlocal terms on the boundary as well.

1. Introduction. In the design of active controllers for stability argumentation of flexible structures, it is naturally important to have a model for the inherent passive damping already present. Often, it is assumed that the damping in each mode is strictly proportional to the mode frequency. If, as in [1], we formulate the problem in the abstract wave equation form as

$$M\ddot{x}(t) + Ax(t) + 2\zeta D\dot{x}(t) + Bu_c(t) = 0,$$

for strictly proportional damping, it is necessary that (see [1])

$$D = \sqrt{M}\sqrt{T}\sqrt{M},$$

where

$$T = \sqrt{M^{-1}}A\sqrt{M^{-1}}.$$

Usually, M commutes with A so that

$$D = \sqrt{M}\sqrt{A}.$$

The problem, then, is of calculating $\sqrt{A}$ in each concrete case which turns out to be nontrivial, in general, even though the theory is well developed [4, 5]. In this paper, we present an explicit calculation of the square root for the case of beam

Received June 16, 1989.
†Research supported in part under Grant No. 88-0252, AFOSR, USAF.
AMS Subject Classifications: 35M05, 76A10.