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FINITE RANK, RELATIVELY BOUNDED PERTURBATIONS OF SEMI-GROUPS GENERATORS. PART III: A SHARP RESULT ON THE LACK OF UNIFORM STABILIZATION

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Abstract. We provide a result of negative character on the lack of uniform stabilization when a group generator A on a Hilbert space Y is additively perturbed by a (typically nondissipative) perturbation operator which is A-bounded and of finite rank (range). Applications include conservative systems (waves and plate equations) with feedback operators on the boundary of the spatial domain. Such result is sharp in two directions: (i) within the class of finite range perturbations, and (ii) within the class of A-bounded perturbations. Indeed, uniform (exponential) stabilization may indeed occur in the first case (i), provided that the perturbation is just "ε worse" than A-bounded (constructive example of an hyperbolic mixed problem with boundary damping provided); and in the second case (ii), provided that the perturbation is A-compact rather than just finite rank (constructive example of an elastic system provided).

1. Introduction, summary of results, literature. Let Y be a (separable) Hilbert space with inner product $(\ ,\ )$. Let $A : Y \supset \mathcal{D}(A) \to Y$ be a (closed, densely defined) linear operator, which is assumed to be the generator of a s.c. (strongly continuous) semigroup or group of operators on Y, conveniently denoted by $\exp[At]$. Let a and b be two arbitrary vectors in Y and consider the abstract dynamics

$$\dot{y} = Ay + (Ay, a)b, \quad y(0) = y_0 \in Y \tag{1.1}$$

$$A_F \equiv A + (A\cdot, a)b, \quad \mathcal{D}(A_F) \supset \mathcal{D}(A) \tag{1.2}$$

$$P \equiv (A\cdot, a)b. \tag{1.3}$$

We note that the perturbation operator P is A-bounded (or relatively bounded with respect to A, [15], one-dimensional range[1], and, typically, non-dissipative. Thus,

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[1]For simplicity of writing we take one-dimensional range perturbations, even though the main results here plainly extend to finite rank (range) perturbations.