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ON PERIODIC SOLUTIONS OF THE NONLINEAR SUSPENSION BRIDGE EQUATION

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Introduction. In this paper we investigate nonlinear oscillations in the nonlinear suspension bridge equation, in $(-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}$, of the type

$$\begin{aligned} &-K_1 u_{xxtt} + u_{tt} + K_2 u_{xxxx} + K_3 u^+ = 1 + k\cos x + \epsilon h(x,t), \\ &u(\pm\frac{\pi}{2}, t) = u_{xx}(\pm\frac{\pi}{2}, t) = 0. \end{aligned} \tag{0.1}$$

The first term in (0.1), due to L. Rayleigh, represents the effect of rotary inertia, as can be traced from the derivation. In many applications, its effect is small.

McKenna and Walter [6] studied nonlinear oscillations in a nonlinear suspension bridge equation without the first term in (0.1):

$$\begin{aligned} &u_{tt} + u_{xxxx} + bu^+ = 1 + \epsilon h(x,t) \ \text{ in } \ (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R} \\ &u(\pm\frac{\pi}{2}, t) = u_{xx}(\pm\frac{\pi}{2}, t) = 0. \end{aligned} \tag{0.2}$$

This equation represented a bending beam supported by cables under a constant load $w = 1$. The constant b represented the restoring force if the cables were stretched. The nonlinearity u^+ models the fact that cables resist expansion but do not resist compression. They proved a counterintuitive result: if the cables were weak; that is, b is small, then there was only a unique solution. However, if b was large (that is, the cables were strengthened), then large scale oscillatory periodic solutions existed.

In this paper we improve this result in two ways. First, we generalize the beam equation to include the effect of rotary inertia. Second, we allow b to vary with x, as indeed it must be suspension bridges.

In Sections 1 and 2 we shall deal with the nonlinear bridge equation with constant coefficients, in $(-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}$

$$\begin{aligned} &-\tfrac{1}{4} u_{xxtt} + u_{tt} + u_{xxxx} + bu^+ = 1 + k\cos x + \epsilon h(x,t) \\ &u(\pm\frac{\pi}{2}, t) = u_{xx}(\pm\frac{\pi}{2}, t) = 0, \end{aligned} \tag{0.3}$$

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