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DISSIPATIVITY CONDITIONS AND VARIATIONAL PRINCIPLES FOR THE HEAT FLUX EQUATION WITH MEMORY†

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Abstract. New restrictions, which turn out to be necessary and sufficient conditions for the second law of thermodynamics for cyclic processes, are found for the constitutive equations of a rigid heat conductor with linear memory. In addition, a stationary principle is given for the corresponding integrodifferential problem and it is shown that the previous restrictions allow one to formulate a minimum principle for it.

0. Introduction. In recent times various authors (see [4], [5], [10]) have studied the heat conduction problem in rigid bodies characterized by linear constitutive equations with fading memory. In this paper, by making use of the second law of thermodynamics in the form given by Clausius, we get new restrictions on the constitutive equations of such materials and, more precisely, on the Fourier and Laplace transforms of their integral kernels. Such restrictions are useful because, at least for linear materials, they are necessary and sufficient for the validity of the second law, as in the viscoelastic and the electromagnetic cases ([11], [13]), where the analogous restrictions on the constitutive relations turn out to be equivalent to thermodynamic principles.

In Sections 5 and 6 we consider the mixed problem of the heat conduction in a rigid conductor with linear memory. For this problem we first establish a stationary principle by making use of the Laplace transformed problem; we then verify that the restrictions placed by the second law on the constitutive equations imply that the Laplace transform of the solution of the mixed problem minimizes a family of functionals and vice versa.

Finally, by following a method of Riess ([7], [9], [12]), we find a class of functionals, depending on a weight function, for each of which it is possible to establish a minimum principle.

1. Preliminaries. Consider a rigid heat conductor occupying a fixed and bounded domain Ω in the Euclidean three-dimensional space. Fixing our attention on a specific point $\mathbf{x} \in \Omega$, we let $\theta(t)$ and $\mathbf{g}(t) = \nabla\theta(t)$ be the absolute temperature and its gradient at $\mathbf{x}$.

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