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PRINCIPAL PAIRS OF SOLUTIONS OF LINEAR SECOND ORDER OSCILLATORY DIFFERENTIAL EQUATIONS

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Abstract. For nonoscillatory second order linear differential equations W. Leighton, M. Morse and P. Hartman introduced almost fifty years ago the notion of *principal solution*–a certain exceptional solution with significant consequences in the description of asymptotic behaviour of solutions. In the oscillatory case, however, such a definition cannot be applied. Guided by the exceptional properties of the pair $\sin t$, $\cos t$ of solutions of $y'' + y = 0$ and similar properties of certain pairs of special functions (e.g., the pair $J_\nu(t)$, $Y_\nu(t)$ of solutions of Bessel equation), the paper offers definitions of "good" pairs of solutions of linear second order differential equations in the oscillatory case. The *principal* pairs and *extremal* pairs are introduced here and conditions for their existence and coincidence as well as their basic properties are derived.

1. Introduction. For nonoscillatory second order linear differential equations of the form

$$y'' + q(t)y = 0 \qquad \text{on} \qquad [a, \infty) \tag{1.1}$$

W. Leighton, M. Morse and P. Hartman (see [6], Chapt. XI, 6) introduced a special, principal solution y_1 defined (up to a constant factor uniquely) by the requirement

$$\int^{\infty} y_1^{-2}(t)dt = \infty,$$

which is equivalent to the condition that

$$\lim_{t\to\infty} \frac{y_1}{y_2} = 0$$

for any solution y_2 of (1.1) linearly independent of y_1.

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