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EXISTENCE OF POSITIVE SOLUTIONS FOR SEMILINEAR ELLIPTIC EQUATIONS IN ANNULAR DOMAINS*

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Dedicated to the memory of Peter Hess

1. Introduction. During recent years much work has been devoted to the study of positive solutions of semilinear elliptic equations where the nonlinear terms grow superlinearly at infinity. Many results exist which show that depending upon the topology of the underlying domain (see [1, 5]), different existence results must be expected, and in fact, existence of positive solutions on annular domains hold for larger classes of nonlinearities rather than for the case where the domain is a ball.

In this paper we continue the study of such problems on annular domains and provide a somewhat unified treatment of earlier results. The dependence of the nonlinear terms upon the independent variable will be assumed radial and we shall consider (most of the time) the existence of radial solutions. Since radial solutions are solutions of an associated boundary value problem for ordinary differential equations, most of our paper will relate results about such problems and will hence also deviate somewhat from the special form of the equation obtained from the partial differential equation when looking for radial solutions.

Thus, let us consider the problem

$$\Delta u + f(|x|, u) = 0, \ 0 < r < |x| < R \tag{1.1}$$

$$u = 0, \ |x| = r, R, \tag{1.2}$$

where $f : [r, R] \times \mathbb{R} \to \mathbb{R}$ is a continuous function and $x \in \mathbb{R}^n$, with $|x|$ the Euclidean norm of x. If u is a solution of (1.1) which only depends upon the radial variable $t = |x|$ then u solves the ordinary differential equation ($' = \frac{d}{dt}$)

$$u'' + \frac{n-1}{t}u' + f(t, u) = 0, \quad r < t < R. \tag{1.3}$$

In this paper we shall consider, for continuous functions a, b, the boundary value problem

$$u'' + a(t)u' + b(t)|u'| + f(t, u) = 0, \quad u(r) = u(R) = 0, \quad r < t < R \tag{1.4}$$

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