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ON THE SUM OF MAXIMAL MONOTONE OPERATORS AND AN APPLICATION TO A NONLINEAR INTEGRO-DIFFERENTIAL EQUATION

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1. Introduction. This paper deals with the sum of a linear ($\mathcal{L}$) and a nonlinear ($\mathcal{A}$) maximal monotone operator in a Hilbert space. We prove that, under certain conditions, the sum $\mathcal{L} + \mathcal{A}$ with domain $D(\mathcal{L} + \mathcal{A}) = D(\mathcal{L}) \cap D(\mathcal{A})$ is maximal monotone as well. As an application we prove existence and uniqueness of a strong solution (see Section 3) of nonlinear integro-differential equations of the form

$$\begin{cases} \dfrac{d}{dt}\displaystyle\int_0^t k(t-s)u(s)\,ds + Au(t) \ni f(t), \quad t \in (0,T); \\ u(0) = 0, \end{cases} \tag{1}$$

where A is a nonlinear, possibly multivalued, maximal monotone operator in a real Hilbert space $(H, \langle\cdot,\cdot\rangle, |\cdot|)$ normalized by $0 \in A0$, $k \in L^1(0,T)$ a real-valued function and $f \in L^2(0,T;H)$ a given function.

As a motivation for studying equations of the form (1), consider the nonlinear Volterra equation

$$u(t) + \int_0^t b(t-s)Au(s)\,ds \ni u_0 + \int_0^t b(t-s)f(s)\,ds, \quad t \in [0,T], \tag{2}$$

where $b \in L^1(0,T)$, $u_0 \in H$ and $f \in L^2(0,T;H)$. As a general reference on Volterra equations we mention Gripenberg et al. [19]. Equations of the type (2) have been studied first by Barbu [3] and Londen [23] where A is the subdifferential of a convex function. The case where A is maximal monotone, not necessarily a subdifferential, has been considered by Gripenberg [17]. The equation in the Banach space setting, that is, equation (2) with A an m-accretive operator in a Banach space X, $u_0 \in X$ and f a function taking values in X, has been treated by Crandall and Nohel [14] and more recently by Gripenberg [18].

Motivated by positivity preserving and invariance properties for equation (2) in an ordered Banach space, Clément and Nohel [11] introduced a class of kernels b, called *completely positive* in Clément and Nohel [12]. By [12, Theorem 2.2], a kernel

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