Differential and Integral Equations, Volume 7, Number 6, November 1994, pp. 1573 – 1596.

THE PRINCIPLE OF LINEARIZED INSTABILITY FOR A CLASS OF EVOLUTION EQUATIONS

B. SCARPELLINI

Mathematisches Institut, University of Basel, Rheinsprung 21, CH-4051 Basel, Switzerland

In memoriam Peter Hess

Abstract. In this paper we investigate instability properties of reaction diffusion systems of the form

$$u_t = D\Delta u + F(u), \tag{*}$$

where D is a diagonal matrix with positive entries, $F(u)$ a polynomial nonlinearity, $u = (u_1, \dots, u_n)$ an n-vector with components in some Sobolev space and Δ the Laplacian on $\mathbb{R}^m$, $m \leq 3$, acting componentwise on u. It is assumed that a smooth, L-periodic equilibrium solution $v = (v_1, \dots, v_n)$ of $(*)$ is given. The instability of v is investigated, not with respect to L-periodic perturbations, as it is usually done, but rather with respect to perturbations in $(H^2(\mathbb{R}^m))^n$. The principle of linearized instability is proved: if the spectrum of $D\Delta + dF(v)$, considered as an unbounded operator in $(H^2(\mathbb{R}^m))^n$, contains a point $\lambda \in \mathbb{C}$ with $\mathrm{re}(\lambda) > 0$ (not necessarily an eigenvalue!) then v is unstable against perturbations in $(H^2(\mathbb{R}^m))^n$.

I. Introduction. In this paper we establish the principle of linearized instability for a class of evolution equations

$$u_t = Au + F(u), \tag{*}$$

to be described below. Here A generates a holomorphic semigroup and F is a nonlinearity satisfying $F(0) = 0$. That is, we prove for this class $(**)$ if $\lambda \in \sigma(A)$ and $\mathrm{re}(\lambda) > 0$ for some λ then the equilibrium solution $u \equiv 0$ of $(*)$ is unstable. In case where A has compact resolvents, $(**)$ is proved in D. Henry [3]; more generally, if the set $\{\lambda : \lambda \in \sigma(A) \text{ and } \mathrm{re}(\lambda) > 0\}$ is $\neq \emptyset$ and compact, $(**)$ is proved in H. Kielhöfer [5]. For A selfadjoint, without any restriction on the spectrum, $(**)$ was proved in [11]. In the first two cases, a suitable projection operator was constructed with the aid of certain resolvent integrals, while in the third case, the projection operator was extracted from the spectral composition of A. In the case to be treated here, A is (in general) not selfadjoint and the set $\{\lambda : \lambda \in \sigma(A)$ and $\mathrm{re}(\lambda) > 0\}$ is not compact. Here the required projection operator is constructed via

Received August 1993.

AMS Subject Classifications: 35B10, 35B20, 35B35, 35K22, 35K57.