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A PARAMETER DEPENDENT TIME-PERIODIC REACTION-DIFFUSION EQUATION FROM CLIMATE MODELING: S-SHAPEDNESS OF THE PRINCIPAL BRANCH OF FIXED POINTS OF THE TIME-1-MAP

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Dedicated to the memory of Peter Hess

Abstract. In this note we establish under two structural hypotheses that the principal branch of fixed points of the time-1-map of a periodically forced, parameter dependent reaction-diffusion equation is S-shaped. The structural assumptions are quite similar to those used in [8] for the time-autonomous case. The problem arises from climate modeling, and including periodic forcing is motivated by the seasonal variation of the incoming solar radiation flux.

1. Introduction. We are concerned with the principal branch $\mathfrak{P}$ of $\mathfrak{S} := \{(\mu, w) \in \mathbf{R}_+ \times Y : \Pi(\mu, w) = w\}$, where Π denotes the time-1-map of

$$\begin{aligned} & c(x)\, \partial_t u(t, x) - \operatorname{div}(k(\cdot) \operatorname{grad} u(t, \cdot))(x) \\ &= \mu Q(t, x)\,[1 - \alpha(x, u(t, x))] - g(u(t, x)) \end{aligned} \tag{1μ}$$

and Y:= $C^+(M)$ with M a two-dimensional oriented compact Riemannian manifold. In carrying over results from the stationary case (cf. [8]), when $Q = Q(x)$, we establish under analogous structural hypotheses that $\mathfrak{P}$ is the trace of an S-shaped simple curve, which connects $(0, \mathbf{0})$ with (∞, ∞).

In case that M $= S^2$, (1μ) arises from so-called energy balance climate models (cf. [5], [9]), which describe the evolution of a long-term mean of atmospheric temperature u. The horizontal heat transport is roughly incorporated by a diffusive approximation. Considering Q as a function of position x alone, means that we are dealing with a yearly mean of temperature, and then it is natural to assume that $\inf Q > 0$. However, when including the seasonal variation ($Q = Q(t, x)$, $Q(\cdot, x)$ 1-periodic), the case we are going to deal with, such a hypothesis is no longer justified in view of the polar night. From a mathematical point of view this is the major extra difficulty as compared with [8] or [10].

The lecture notes volume of Peter Hess [7] provides an excellent survey on recent progress in studying time-periodic semilinear parabolic boundary value problems. It

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