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# ON QUASILINEAR FULLY PARABOLIC BOUNDARY VALUE PROBLEMS

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In memoriam Peter Hess

**1. Introduction.** In the present paper we discuss some recent results concerning quasilinear parabolic boundary value problems. We shall prove existence and uniqueness of solutions of such systems. Moreover, it will be shown that the corresponding solutions generate a smooth local semiflow on suitable Banach spaces including the standard Sobolev spaces $W_p^1(\Omega)$.

Generally, quasilinear systems contain a copious (technical) structure. Thus, for the sake of simplicity, we first consider a model case to describe the main ideas. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a smooth boundary $\Gamma = \partial\Omega$. We are looking for two functions $u_i : \overline{\Omega} \times [0, \infty) \to \mathbb{R}$, $i = 1, 2$, satisfying the following equations:

$$\begin{aligned} \partial_t u_1 - \Delta p_1(u_1, u_2) &= f_1(u_1, u_2) \quad \text{in } \Omega \times (0, \infty), \\ \partial_t u_2 - \Delta p_2(u_1, u_2) &= f_2(u_1, u_2) \quad \text{in } \Omega \times (0, \infty), \\ \partial_t u_1 + \partial_\nu p_1(u_1, u_2) &= g_1(u_1, u_2) \quad \text{on } \Gamma \times (0, \infty), \\ \partial_t u_2 + \partial_\nu p_2(u_1, u_2) &= g_2(u_1, u_2) \quad \text{on } \Gamma \times (0, \infty). \end{aligned} \tag{1.1}$$

We assume that $p_r$, $f_r$, and $g_r$ are given smooth functions, i.e.,

$$p_r,\ f_r,\ g_r \in C^2(\mathbb{R}^2, \mathbb{R}), \qquad r = 1,\ 2.$$

Moreover, $\Delta := \sum_{j=1}^n \partial_j^2$ denotes the Laplace operator in Euclidean coordinates and $\partial_\nu := \sum_{j=1}^n \nu^j \partial_j$ stands for the derivative with respect to the outer normal $\nu = (\nu^1, \dots, \nu^n)$ on $\Gamma$.

Let us briefly discuss the equations appearing in (1.1). Under suitable structural conditions for $p_1$ and $p_2$, see (1.2), (1.3), the first two equations in (1.1) are a simple example of a *strongly coupled quasilinear parabolic system* in the domain $\Omega$.

It is worthwhile to remark that the second two equations in (1.1) describe *non-standard* boundary conditions, since they involve besides spatial derivatives also

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