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ON THE FIRST CURVE OF THE FUČIK SPECTRUM OF AN ELLIPTIC OPERATOR

D.G. DE FIGUEIREDO

IMECC, UNICAMP, Caixa Postal 6065, 13081 Campinas-SP ,Brazil

J.-P. GOSSEZ

Département de Mathématique, C.P.214, Université Libre de Bruxelles, 1050 Bruxelles, Belgium

Dedicated to the memory of Peter Hess

Abstract. We obtain a variational characterization of the first nontrivial curve in the Fučik spectrum of the Laplacian. We study the asymptotic behavior of this first curve and exhibit a connection with the antimaximum principle. We also obtain a nodal domain theorem for the corresponding eigenfunctions. An application to the study of nonresonance is given.

1. Introduction. The Fučik spectrum of $-\Delta$ on $H_0^1(\Omega)$ is defined as the set Σ of those $(\lambda_+, \lambda_-) \in \mathbb{R}^2$ such that

$$-\Delta u = \lambda_+ u^+ - \lambda_- u^- \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega \tag{1.1}$$

has a nontrivial solution u. Here $u^+ = \max(u, 0)$, $u^- = \max(-u, 0)$ and Ω is a bounded domain in $\mathbb{R}^N$, $N \geq 1$. Denoting by $\lambda_1 < \lambda_2 < \dots$ the distinct eigenvalues of $-\Delta$ on $H_0^1(\Omega)$, clearly Σ contains each (λ_k, λ_k) and the two lines $\lambda_1 \times \mathbb{R}$ and $\mathbb{R} \times \lambda_1$.

Several works have been devoted to the study of Σ. Of particular interest for our purposes are those of [6] where it is shown that the two lines $\lambda_1 \times \mathbb{R}$ and $\mathbb{R} \times \lambda_1$ are isolated in Σ, and [12], [18], [17] where curves in Σ emanating from each (λ_k, λ_k) are shown to exist locally. In fact, combining the analysis in [17] with the isolation result of [6], one can show that Σ contains a first curve through (λ_2, λ_2) which extends to infinity (cf. also [16]).

It is our purpose in this paper to prove directly the existence of such a first curve and to give a variational characterization of it. Drawing a line of positive slope from (λ_1, λ_1), we obtain its first intersection point with Σ through some constraint minimization of a Dirichlet type integral. This construction was motivated by a previous work of one of us with M. Cuesta relative to the case $N = 1$ (cf. [5]). The formula for the first curve C_1 that we obtain in this way provides a natural extension

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