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NON EXISTENCE AND OTHER PROPERTIES FOR SOLUTIONS OF QUASILINEAR ELLIPTIC EQUATIONS

Roberta Filippucci and Patrizia Pucci
Dipartimento di Matematica, Università degli Studi di Perugia, Perugia, Italy

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1. Introduction. In 1986, Ni and Serrin established some existence and non existence theorems for radial *ground state solutions* of the partial differential equation

$$\mathrm{div}(A(|Du|)Du) + f(u) = 0, \tag{1.1}$$

where Du denotes the gradient of u,

$$f \in C(\mathbb{R}), \quad f(0) = 0, \quad f(u) \geq 0 \quad \text{near } u = 0,$$

and either A comes from the degenerate Laplace operator, namely $A(|p|) = |p|^{m-2}$, $m > 1$, or

$$A \in C^1(\mathbb{R}^+;\mathbb{R}^+), \quad A(p) = O(1) \quad \text{as } p \to 0^+, \tag{1.2}$$

as for the mean curvature operator. Of course (1.1) reduces to a semilinear Laplace equation when $A \equiv 1$.

Specifically, a ground state is a nonnegative, nontrivial solution u of (1.1) in $\mathbb{R}^n$, $n > 2$, which tends to zero at ∞. Since one can generally expect ground states to be radially symmetric, the main results of [2] are based on the study in $\mathbb{R}^+$ of the ordinary differential equation

$$[t^{n-1}A(|u'|)u']' + t^{n-1}f(u) = 0, \qquad t = |x| \in \mathbb{R}^+ \qquad ('= d/dt). \tag{1.3}$$

The purpose of this paper is to extend in a unified form the non-existence theorems of [2] to general variational equations of the form

$$[g(t)G_p(u')]' + g(t)f(t,u) = 0, \tag{1.4}$$

where

$$\left.\begin{array}{l} G(p) \in C^1(\mathbb{R}), \;\; G \text{ is strictly convex in } \mathbb{R}, \;\; G(0) = G_p(0) = 0, \\ G_p(p) = O(|p|^{m-1}) \;\text{ as } p \to 0^+, \;\; m > 1; \\ f \in C(\mathbb{R}_0^+ \times \mathbb{R}), \;\; \text{and } \; F(t,u) = \int_0^u f(t,v)dv \;\text{ is of class } \; C^1(\mathbb{R}_0^+ \times \mathbb{R}); \\ g \in C^1(\mathbb{R}^+) \cap C(\mathbb{R}_0^+), \;\; g(0) = 0, \\ g > 0, \;\; \delta = g'/g \geq 0 \text{ in } \;\mathbb{R}^+. \end{array}\right\} \tag{I}$$

Indeed (1.3) is the special case of (1.4) when $g(t) = t^{n-1}$ and $G(p) = \varphi(|p|)$, so that $A(|p|) = \varphi'(|p|)/|p|$. For the degenerate Laplace operator, namely $G(p) = |p|^m/m$,

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