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ON VERY SINGULAR SELF-SIMILAR SOLUTIONS FOR THE POROUS MEDIA EQUATION WITH ABSORPTION

Giovanni Leoni
Dipartimento di Matematica, Università di Perugia, via Vanvitelli 1, 06123 Perugia, Italy

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1. Introduction. In a recent paper ([12]) we have studied the existence of global solutions of strongly degenerate quasi-variational ordinary differential systems of general form. The canonical model is given by

$$\big(g(t)\mathcal{L}_v(u,u')\big)' = g(t)\big\{\mathcal{L}_u(u,u') + Q(t,u,u')\big\}, \tag{1.1}$$

where

$$\mathcal{L}(u,v) = A(u)|v|^s - F(u), \qquad s>1,$$

with $A(u)>0$ for $u \neq 0$ (and possibly $A(0)=0$), g is a nondecreasing function and the damping term Q has the form $Q(t,u,v) = -c\,t^{\gamma}|u|^{\alpha}|v|^{\beta}v$, with α possibly negative.

Systems of this kind arise naturally from degenerate elliptic systems, with or without a gradient term, as well as from degenerate parabolic equations. The simplest general examples are the porous media equation

$$w_\tau = \triangle(w^m) \qquad \text{in } \mathbb{R}^n \times \mathbb{R}^+ \tag{1.2}$$

and the porous media equation with absorption

$$w_\tau = \triangle(w^m) - w^p \quad \text{in } \mathbb{R}^n \times \mathbb{R}^+, \qquad p > \max\{m,1\}, \tag{1.3}$$

where $w = w(x,\tau)$, $n \geq 1$ and, as usual, $\mathbb{R}^+ = (0,\infty)$. These equations have been treated extensively in the literature, both in the *slow diffusion case*, that is, $m>1$, and in the *fast diffusion case*, that is, $0<m<1$. See, for example, the work of Aronson and Peletier; Bénilan, Crandall and Pierre; Diaz and Saa; Gilding and Peletier; Herrero and Pierre; Kamin and Peletier; Kamin, Peletier and Vazquez; Kamin and Veron; Peletier and Junning; Peletier and Terman and the references contained therein. When $m=1$ equations (1.2) and (1.3) reduce respectively to the heat equation and the heat equation with absorption.

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