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EXISTENCE AND NONEXISTENCE FOR THE EXTERIOR DIRICHLET PROBLEM FOR THE MINIMAL SURFACE EQUATION IN THE PLANE

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1. Introduction and statement of main results. The aim of this paper is to investigate the classical solvability of the Dirichlet problem for the minimal surface equation in the plane

$$Mu := (1+|Du|^2)Du - D_iuD_juD_{ij}u = 0 \quad \text{in} \quad \Omega \tag{1}$$
$$u = f \quad \text{on} \quad \partial\Omega, \tag{2}$$

where $\Omega \subset \mathbb{R}^2$ is a domain of class C^2 with bounded complement $\mathbb{R}^2\backslash\Omega$, and the boundary data f are continuous. As is well known from classical results (see [11], Chapter VII) the Dirichlet problem (1), (2) in bounded domains $\Omega \subset \mathbb{R}^2$ is solvable for all data f if Ω is convex and in the multidimensional case if Ω is mean-convex ([3]). Already at the beginning of this century, Korn ([4]) and Müntz ([10]) proved by the method of successive approximations that, in the case of arbitrary bounded domains of class $C^{2,\alpha}, 0 < \alpha < 1$, the smallness of the $C^{2,\alpha}$-norm of the data guarantees the solvability of (1), (2) (see also [11], § 412–414.) Using Perron's method, Nitsche showed for bounded domains of class $C^{1,1}$ and data $f \in C^{1,1}$ that (1), (2) is solvable provided that osc$f := \max f -$ min f is small enough depending on Ω and the $C^{1,1}$-norm of f ([11], §649). A similar result (replacing $C^{1,1}$ by C^2)

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