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# ON THE BERNOULLI FREE-BOUNDARY PROBLEM

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**Introduction.** We consider a bounded domain $\Omega$ in $\mathbb{R}^n$, partially filled with water. The water occupies a fixed part of the domain at an initial time, while the rest is filled with gas of constant pressure. Given some initial velocity distribution we ask how the fluid and the shape of the gas bubble evolve in time.

This problem was considered by Chen and Friedman in [1]. They investigated the quasi-stationary case, i.e., the situation where the fluid velocity does not depend on time. They proved the existence of a unique classical solution local in time and space.

By adapting the weaker notion of a "generalized geometric evolution" introduced by Giga and Takahashi in [2] we will be able to prove an existence result which is global in time. However we lose the information, whether the volume of the gas bubble remains constant during the evolution. The reason is that we can not exclude that the interface has positive two-dimensional Hausdorff measure during some time interval.

We wish to mention a different approach by Nouri and Poupaud [6], where a global weak solution for a multifluid flow was constructed, avoiding the effect of interface fattening.

**1.** We introduce some idealizations in assuming that the fluid is ideal, incompressible and irrotational. Thus we describe the fluid by a harmonic potential $u$ and by the variable $p$ for the pressure. Effects due to surface tension on the interface between the fluid and the gas are neglected. As a consequence the pressure is continuous across the boundary. Since the interface is an unknown itself, we have to find an analytical description for it. We assume that the interface moves with the fluid; i.e., if its velocity is

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