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SCHRÖDINGER EQUATION WITH CRITICAL SOBOLEV EXPONENT

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1. Introduction

In this paper we study the existence of solutions and their concentration phenomena of a singularly perturbed semilinear Schrödinger equation with the presence of the critical Sobolev exponent, that is,

$$\begin{cases} -\varepsilon^2 \Delta u + V(x)u = K(x)u^p + Q(x)u^\sigma & \text{in } \mathbb{R}^N, \\ u > 0 & \text{in } \mathbb{R}^N, \\ \lim_{|x|\to\infty} u(x) = 0, \end{cases} \tag{1.1}$$

where $N \geqslant 3$, $1 < p < \sigma = \frac{N+2}{N-2}$, and V, K, and Q are C^2 functions from $\mathbb{R}^N$ to $\mathbb{R}$. We will show that there exist solutions of (1.1) concentrating near the maximum and minimum points of an auxiliary functional which depends only on V, K, and Q.

On the potentials, we will make the following assumptions:

(V) $V \in C^2(\mathbb{R}^N, \mathbb{R})$, V and D^2V are bounded; moreover,

$$V(x) \geqslant C > 0 \quad \text{for all } x \in \mathbb{R}^N.$$

(K) $K \in C^2(\mathbb{R}^N, \mathbb{R})$, K and D^2K are bounded; moreover,

$$K(x) \geqslant C > 0 \quad \text{for all } x \in \mathbb{R}^N.$$

(Q) $Q \in C^2(\mathbb{R}^N, \mathbb{R})$, Q and D^2Q are bounded; moreover, $Q(0) = 0$.

We point out that while V and K must be strictly positive, Q can change sign and must vanish at 0.

Let us introduce an auxiliary function which will play a crucial rôle in the study of (1.1). Let $\Gamma\colon \mathbb{R}^N \to \mathbb{R}$ be a function so defined:

$$\Gamma(\xi) = \bar{C}_1 \Gamma_1(\xi) - \bar{C}_2 \Gamma_2(\xi), \tag{1.2}$$

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